NUMBERS COUNT

Kaprekar Numbers

New readers start here. The topics dealt with in this column attempt to reach the frontiers of knowledge in number theory with the minimal background information. The problems posed therefore have no complete solution known to the author, and readers are encouraged to submit their attempts at solution, however incomplete they may seem.

(1) The legendary number theorist, D R Kaprekar, of the Indian Institute of Science at Bangalore, is perhaps most famous for publicising the number 6174, which is the eventual result *(with certain exceptions)* of the subtraction of a four digit integer whose digits are arranged in ascending order from the same integer whose digits are arranged in descending order.

For example, given the four digit integer 8923 we proceed thus:
9832 − 2389 = 7443,
9963 − 3699 = 6264,
7641 − 1467 = 6174,
7443 − 3447 = 3996,
6642 − 2466 = 4176, the process repeats at this stage.

Question A. What are the certain exceptions referred to above?

Question B. What happens to integers with other than four digits?

(2) Kaprekar Numbers, however, are defined to be those n-digit integers, K, which are equal to the sum of the integer defined by the least significant (right-most) n-digits of their square plus the integer defined by the remaining digits.

Thus 142857 is a Kaprekar Number because here n = 6 and
(i) $(142857)^2 = 20408\underline{122449}$
(ii) 122449 + 20408 = 142857

Question C. What are the Kaprekar Numbers less than 10^n for a given n?

Interest Note. The square of any cyclic permutation of a K-Number is also a cyclic permutation when the digits are added as required.

For example, $(428571)^2$ = 183673102041 and 102041 + 183673 = 285714.

Readers are invited to submit a program, or suite of programs, to answer the above questions. All submissions should include program listings, hardware descriptions, run times and output; they will be judged for accuracy, originality and efficiency (not necessarily in that order).

A prize of £10 will be awarded to the 'best' entry received. Please address all entries, to arrive by 1 April, to Mr M R Mudge, Room 560/A, Department of Mathematics, University of Aston in Birmingham, Gosta Green, Birmingham B4 7ET.

Review—September 83

The Partitions of a Positive Integer generated a very heavy, and varied response; the extremes being typified by an estimate of 15 years to find p(100) using a TI59 programmable calculator to an offer to calculate p(535) programming in Algol 68 — on an unspecified mainframe, one suspects.

A reference work in this field is provided by GE Andrews, *The Theory of Partitions, Encyclopedia of Mathematics and its Applications,* Volume 2, Addison-Wesley 1976. However, the presentation in Chapter XIX of the fifth edition of *An Introduction to the Theory of Numbers, by* GH Hardy and EM Wright, Oxford University Press 1979 is adequate to yield the recurrence relationship for p(n) without which realistic computing is virtually impossible.

$p(n) - p(n-1) - p(n-2) + p(n-5) + \ldots + (-1)^k\ p(n-\tfrac{1}{2}k(3k-1)) + (-1)^k\ p(n-\tfrac{1}{2}k(3k+1)) \ldots = 0$

To estimate the magnitude of the computation one may use the asymptotic formula of Hardy and Ramanujan 1917,

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

The computational difficulties are referred to in *Computers in Number Theory* edited by AOL Atkin and BJ Birch, Academic Press 1971.

The prizewinning entry is from RB Shepherd of 2 Orchard Croft, Cottingham, Humberside HU16 4HG using Pro-Pascal on a Sharp MZ80-B computer (64 kbytes). This submission factorised up to p(300) in 44hours 20mins. It must be observed that the presentation of results by RB Shepherd, and indeed by numerous other contributors, was of the highest possible standard. Congratulations all round, and keep the entries flowing.

Note. Submissions will only be returned if a suitable stamped addressed envelope is provided.

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LEISURE LINES

by J J Clessa

Quickie

A 20lb monkey hangs from a rope which passes over a single pulley and is balanced by a 20lb weight hanging from the other end of the rope. The monkey begins to climb up the rope. Will the 20lb weight go up, go down — or stay still?

Prize puzzle

Three artists — Albert, Brian and Charles — each paint a picture on a square canvas. Brian's picture is seven square feet more than Albert's in area, and seven square feet less than Charles'. All sides have exact measurements. What are the dimensions of the pictures?

November prize puzzle

The puzzle would have presented very little problem for those of you with micros, although a little thought would have saved quite a bit of computer time — especially if you were to generate *all* 10-digit numbers rather than the smallest.

Since the required answer had to be divisible by the integers 1 to 12, it must be a multiple of 12 x 11 x 7 x 5 x 3 x 2 — that is 27720. Hence you could set up a simple loop with increments of 27720. Luckily, the solution is 1 234 759 680, which comes up relatively early in the loop.

You can cut down considerable time by tailoring the loop to skip whenever the second digit is equal to the first.

The winning entry came from D Spencer of Ruislip, Middlesex whose prize is on its way.

To those overseas entrants who were worried that their entries might arrive too late: I can't recall a case in which a late entry was from overseas.

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