NUMBERS COUNT

Number theories

New readers start here. The topics dealt with in this column attempt to reach the frontiers of knowledge in number theory with the minimal background information. The problems posed therefore have no complete solution known to the author, and readers are encouraged to submit their attempts at solution, however incomplete they may seem.

This month we look back and examine some number theoretic results which were making the news around the turn of the century. We ask how these might be established using a digital computer, and in what ways they may be extended.

The results, which require no understanding of mathematics beyond elementary arithmetic, are given in chronological order and readers are invited to respond to some, or all of them!

(a) In 1876 AB Evans found four integers whose sum is a sixth power, and such that the sum of any three is a fifth power.

(b) In 1895 several writers found two integers whose sum, difference and difference of their squares are all twelfth powers.

(c) In 1898 GBM Zerr found six positive integers x_1, x_2, x_3, x_4, x_5 and x_6, such that each, diminished by $(5/2)(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)^5$ becomes a fifth power, and three years later three numbers in arithmetic progression (A, A+d, A+2d, where A is the smallest and d the common difference of arithmetic progression) whose sum is a sixth power.

(d) In 1904 P F Teilhet verified that every integer, A, up to 600, with one exception is a sum of two squares and two positive or zero cubes.

(e) In 1917 R Goormaghtigh stated: 'For A less than 1000000 $A=1+x+x^2+x^3+...+x^m=1+y+y^2+y^3...+y^n$ holds only in two cases, one of which is $31=1+5+5^2=1+2+2^2+2+2^4$.'

Readers are invited to submit a program, or suite of programs, to recreate the news items listed above, and to extend them in any way. Thus in (d) the exception should be displayed and, hopefully, the bound on A significantly extended, while in (e) the second case should be found explicitly and, again, the bound extended on A.

A prize of £10 will be awarded to the 'best' entry received by 1 May, 1984. Please address all entries to Mr M R Mudge, Room 560/A, Department of Mathematics, University of Aston in Birmingham, Gosta Green, Birmingham B4 7ET.

Note. Criteria of judgment include limitations imposed by hardware and programming language chosen, so details of these should be supplied.

Review — October 83

The concept of a *Perfect Number* appears to be well known to many *PCW* readers, hence the peak in response to this article.

The problem of finding the factors of an integer is recognised by many, but few address themselves to it. The use of the Chinese Remainder Theorem may provide a 'best possible' algorithm, but personally I doubt this. Discussions with Dr R Churhouse and Dr AOL Atkin many years ago at the Atlas Computer Laboratories, Chilton, in the presence of my colleague Dr D Ridout, revealed that there was still an unsolved problem in this area.

The Poulet sequence has been displayed by many correspondents — some suggested problems include:

(1) Are all abundant numbers divisible by 15 if they are odd?

(2) Are there any three-ply Perfect Numbers different from 120 and 672?

Therefore, does there exist an n for which $\sigma(n)=kn$, k greater than 2 defines a k-ply Perfect Number.

The winner is Mr J Jones, 33 Vincent Avenue, Nantgylo, Gwent NP3 4PF. He has taken this problem to the limit of his available hardware, changing the programming language on the way.

Once again, it should be observed that many submissions were of the highest possible standards of neatness; however, I claim to recognise valuable work among the other submissions, and ask that you are not put off by the lack of a word processor.

A number ^{m} is said to be:

2-hyper-perfect if $m=2s(m)-1$

3-hyper-perfect if $m+3s(m)-2$; and in general

n-hyper-perfect if $m=n\,s(m)-(n-1)$.

D Minoli has constructed (1980) a list of all n-hyper-perfect numbers (n greater than 1) up to 1500000 using the PDD 11/70 computer.

Can anyone improve upon this situation?

Note. Submissions will only be returned if a suitable stamped addressed envelope is provided.

END

LEISURE LINES

by J J Clessa

Quickie

Of 50 people interviewed 27 liked cricket, 32 liked soccer and five didn't like either. How many people liked both cricket and soccer?

Prize puzzle

The Smiths and Jones families each have three children who work for the local authority. By coincidence, the salaries of the three Smith children are in the same proportions as those of the three Jones children.

Moreover, Albert Smith and Paul Jones have the same salary — as do Mary Smith and Sally Jones. However, Peter Jones earns £387 per month more than Michael Smith. What is the salary of each?

Answers please — postcards or backs of sealed envelopes only — to reach *PCW* not later than last post on 30 March, 1984. Send your entries to *PCW*, March prize puzzle, Leisure Lines, 62 Oxford Street, London W1.

December prize puzzle

Not too difficult to solve analytically, although it's a bit harder to solve by trial and error using a micro. Naturally, it isn't possible to say which of the two scores is for the penalty and which is for the field goal, but the scores must be 4 and 17.

This prohibits scores of:
1 2 3 5 6 7 9 10 11 13 14 15 18 19 22 23 26 27 30 31 35 39 43 and 47 — twenty-four in all.

The winning entry, drawn at random from over one hundred, came from C R Hensler of Letchworth, Herts. Congratulations, Mr Hensler, your prize is on its way.

Don't forget — please send entries on postcards (backs of sealed envelopes will do). Letters are immediately disqualified.

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