Repunits

This month Mike Mudge takes a long look at Repunits and Repdigits

The first part of this month's problem, although very simple to formulate, should encourage the development of certain general integer length arithmetic routines. See, for example, DE Knuth's, *The Art of Computer Programming*, Vol 2, Semi-numerical Algorithms, Addison Wesley, 1969; such algorithms once optimised will prove invaluable in any future empirical number theory.

The second part, somewhat tenuously related to the first, is in response to numerous requests for further problems relating to Prime Numbers; and is an opportunity to mention the possible sinister significance of such numbers in 1984, hinted at by A Berry, *The Daily Telegraph*, 9 January, 1984 together with the paper *The Fascinating Hunt for Prime Numbers* by C Pomerance in *The Scientific American*, December (1982).

1) Defining a Repunit by $R_n = (10^n - 1)/9$, an integer consisting of a string of n 1's. The problem is to factorise R_n completely for a given n. Thus $R_2 = 11$, $R_3 = 3.37$, $R_4 = 101.11$, $R_5 = 271.41$.
2) Noting that Repdigits (defined in the obvious way) other than Repunits are always trivially composite (not prime), it is known that in common with Repunits they can occur as long strings in primes.

Thus R_{317}, 2222222222 2222222222 39, 33333333333333333333 01, 1733333333 3333333333 33, 4444444444 4444444444 51 are all primes!

Furthermore, $10^{564} + 10^{282} - 1$ consisting of 1 followed by 282 0's and 282 9's is also prime.

Find primes containing lengthy Repdigits 5,6,7 and 8.

It is likely that this section will involve considerable library work and hopefully not too much computing, as a variation on the usual balance between these two activities in Numbers Count.

A prize of £10 will be awarded to the 'best' entry received by 1 June, 1984. Please address all entries to Mr MR Mudge, Room 560/A, Department of Mathematics, University of Aston in Birmingham, Gosta Green, Birmingham B4 7ET.

Note. Criteria of judgement include limitations imposed by hardware and the programming language chosen, so details of these should be supplied.

The Persistence of an Integer Review — November 1983

The Persistence of an Integer provided a popular challenge: with typical results to (c) examining powers up to 2^{9764} radix 3 in about 32 hours of Basic on a BBC Micro.

Parts (d) and (e) are still very much closed books and results relating to them would most certainly be of interest to myself and to this month's prizewinner, Mr Alan Prior of 41 Walnut Tree Road, Shepperton, Middlesex.

Alan used Basic on his Sharp MZ-80A with 48k and a 2MHz processor, having first rejected Pascal and Forth: the former due to the limitations of his version; and the latter due to lack of time to become familiar with the language.

In six hours two minutes (and eight seconds) he established 2777777 88888899 as the smallest number with persistence 11 using a program which handles 78 digit integer input and 255 digit integers internally.

Attempts to find the smallest integer with persistence 12 have so far been unsuccessful, although tables of persistence of n for n = (1) 24999, if extended, may shed light on this problem, should some underlying pattern be revealed.

The origins of this problem, to the best of my knowledge, are to be found in NJA Sloane's *The Persistence of a Number, Journal of Recreational Mathematics*, Vol 6 1973 (pp 97-98).

Note. Submissions will only be returned if a suitable stamped addressed envelope is provided.

by J J Clessa

Quickie

If 250 players enter a darts knockout tournament, how many matches will have been played by the time the tournament is finally won?

Prize puzzle

Can you complete the 3×3 grid shown

	C	P	S
S			
P			
C			

here so that the row and column marked 'P' contain a prime number, those marked 'C' contain a perfect cube, and those marked 'S' contain a perfect square.

January prize puzzle

A very mediocre response to the January puzzle, about 50 entries only. Perhaps it was more difficult than usual, or maybe my readers are spending their time on the puzzles to win the Apricot.

However, many of those who did submit entries found quite a lot more to the puzzle than I realised. Clearly, there are many solutions, the smallest of which (with 7 digits) is 1000146, whose divisors total 2286144 (1512^2). The largest is 9998508 which is 5040^2.

The winner was chosen by a draw, and the lucky entrant was from Milan, Italy — Mr Giorgio Vincenti. He only submitted one solution, 1380527, whose divisors sum to 1176^2 but it was enough to win the prize. Congratulations, Giorgio, your prize is on its way.

Incidentally, solutions should always be submitted on postcards or the backs of sealed envelopes. Normally solutions on letters are ineligible for prizes, but since we forgot to state this in the January puzzle we let it go by this once. But don't forget — letters if you want to correspond, postcards for the puzzle entries.