

Tree-like structures

This month Mike Mudge guides you through the ramifications of tree-like structures due to Collatz.

As a student, the famous numerical analyst L Collatz asked if the sequence defined by:

$a_{n+1} = a_n / 2$ when a_n is even together with $a_{n+1} = 3a_n + 1$ when a_n is odd, is tree-like apart from the root 4,2,1,.

By this he meant, starting from an arbitrary positive integer a_1 and repeatedly applying the appropriate formula chosen from the above pair, is there always a value of n , which we may denote by $c(a_1)$ such that $a_n = 1$?

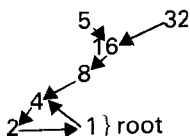
For example: if $a_1 = 10$, then $a_2 \times 5, a_3 = 16, a_4 = 8, a_5 = 4, a_6 = 2, a_7 = 1$. Thus we write $c(10) = 7$. It is readily verified that $c(152) = 24$ also that $c(18) = 21$.

D H & E Lehmer together with J L Selfridge have answered this in the affirmative for 0 less than a_1 less than 10^9 while as recently as 1978 Corroda Böhm and Giovanna Sontacchi extended the range of a_1 up to 7×10^{11} .

Now if $3a_n + 1$ is replaced by $3a_n - 1$ it seems likely that the possible trees have roots (1,2,..) or (5,14,7,20,10,..) or (17,50,25,74,37,110,55,164,82,41,122,61,182,91,272,136,68,34,..)

The generalisation of these two problems is due to D C Kay (Pi Mu Epsilon Journal, Vol 5, 1972, page 338) who defines a sequence thus $a_{n+1} = a_n/p$ if p is a factor of a_n else $a_{n+1} = a_n \times q + r$ and asks for what values of p, q and r the problem can be completely analysed?

Readers are invited to reproduce the results given above, and to extend them in any natural way. Particular interest is focused upon computer generated tree-like structures which may help to reveal the underlying behaviour of these Collatz sequences — which it must be emphasised are not one-one. In general we cannot retrace the history of a sequence as the inverse is often not unique — the tree branches!



Submissions should include program listings, hardware descriptions,

run time and output; they will be judged for accuracy, originality and efficiency (not necessarily in that order). A prize of £10 will be awarded to the 'best' entry received by 1 September 1984. Please address entries to Mr MR Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. Telephone comments, both favourable and otherwise, are welcome on (0902) 892141.

Review of Numbers Count -13- Kaprekar Numbers

In addition to numerous regular respondents both from the continent and the UK one submission introduced a new efficiency parameter; being the ratio of computer work time to programming time and estimated at 112hrs. : one evening using Knight's extended

This month's prizewinner is W J Duffin of 14 Orchard Croft, Cottingham, North Humberside, HU16 4HG using Basic on a BBC Micro 32k, OS 1.20, with a Microvitec medium resolution monitor, a Sanyo Slim 3A cassette recorder for saving programs and an MCP-40 printer/plotter or Olympia Compact 2 electronic typewriter for listing and output.

The exceptions in question A are seen to be the trivial case of all digits equal; also the case when the fourth differs by unity from the other three when the first difference is always 999.

It is hoped that the reproduction of Bill Duffin's results to questions B & C, together with his closing comments upon this problem, will encourage other readers to try their hand at 'Numbers'.

Due to lack of space, results to Question C are printed on page 209.

Type of cycle *	Value of n							
	2	3	4	5	6	7	8	9
C(1)		1 (954)	1 (7641)		2 (995544) (766431)		1 (98754210)	
C(2)				1 (95553)				
C(3)							1 (88763220)	
C(4)				2 (97641) (97443)				
C(5)		1 (81)						
(C7)					1 (875421)		1 (87664320)	
C(8)						1 (9775431)		
C(14)								1 (996654321)

Question B: tabulated results

495
6174
549945
631764

For each cycle, one ordered n-digit number within the cycle is given, from 97508421

For example, C(3) for n=8 means that many 8-digit numbers eventually yield which the others can be generated.

Thus the only self-generating numbers under this operation, having one of the differences 86526432, 64308654, 83208762 which regenerate digits or fewer, would seem to be:

86526432, 64308654, 83208762 which regenerate.



TRANSACTION FILE

- TRS-80 Model III. 1 disk drive 48k complete with manuals, Infocast program etc; DMP-100 printer also available. Reasonable offers please write J.R. Mayhew, 5 Bronte Close, Tilbury, Essex RM18 8BH.
- MAPLIN modem. 300 Baud Orig/Answer. Boards professionally assembled and tested but need to be fitted in case. All components and full data, inclusive of postage £40. Tel: (0670) 829215.
- GEMINI G805. Single disk drive (DS/SD 35 track) with CPM and controller for Nascom 1/2 £150. Also disks in plastic boxes £12 per 10. Tel: Ringmer (0273) 813342 (Sussex).
- EPSON RX80 printer. As new and under manufacturers warranty. Also Telexwriter word processing program, to suit Dragon 32 for £195.

- Tel: G. Booth (0843) 43793.
- NEWBRAIN AD with leads and manuals, original packing. Bargain price only £160 ono. J. Andrews. Tel: (021) 354 4047.
- SWTP computer and stand-alone VDU and keyboard. Motorola 6800 based motherboard system. Dual, full control, cassette interface, 24k RAM. Spare processor board. Comprehensive documentation and software. £180. Tel: Wakefield 250646.
- PLUTO colour graphics controller. Extended ROM £250 ono. Nascom 2 48k RAM toolkit, books, magazines, games. Chess etc £130 ono. Tel: Birmingham (021) 429 2420 after 7pm.
- SHARP PC1500 pocket computer, 8k RAM. Plotter printer pens and paper. Hardly used £250

- ono. 01-864 5784 evenings.
- NASCOM I, 64k I/O board, 5000 Baud computer controlled cassette deck, software, much more, £500 ono. With GP80A printer £600. Mr M. Parker, 22 Hutchcomb Road, Botley, Oxford. Tel: (0865) 725495.
- WOULD anyone be willing to donate any computer hardware to a computer club? Write to Towcester Microcomputer Users Group, 20 Wordsworth Close, Towcester, Northants.
- ACT Sirius 1 128k RAM. 1.2Mb floppy disc + Epson EX80. Also listing paper and various software. Just 6 months old £2,500 ono. Tel: (0795) 534772.
- SHARP MZ-700. Built-in tape drive, 64k, can be used with colour TV. Knights software, Forth, Pascal, Machine Code, included. Hardly used £175 ono. Langton, 01-628 4266

- (work).
- OSBORNE IS. 80 column, double density + CP/M, CBasic, MBasic, Wordstar, Supercalc, Mailmerge. Hardly used £900 ono. Tel: (021) 705 9340.
- IBM; model 73 golf ball electric typewriter. Recently overhauled, circuit and mechanical diagrams available for knowledgeable amateur to interface to home computer £100 ono. Tel: (061) 969 7847 (eves).
- SUPERBRAIN computer; 64k 2 x 168k disk drives, reliable machine with CPM programmes, 2 x RS323 interfaces, manuals. Excellent word processor, versatile machine. £750 + VAT. Tel: 01-959 2049.
- ITT2020 (Apple 2), 64k. Apple disc drive, Silentyper printer. Some software, Dos 3.3, Applesaver, Toolkit, Visicalc, etc. £399. Tel: Crewe (0270) 780608.
- PET 3032. With 3022 printer and cassette. Basic 4, Toolkit and Arrow Chips. Very good condition. £550 ono. Winsford (06065) 58083.
- APPLE Graphics Tablet £230. Utopia Tablet software £20. Digitek colour card £40. Bitstik 'Robo 500' £160. Appletworld £10, 3-D Supergraphics £10. Bill Budge's 3-D Graphics system £10. Tel: Tim. 01-263 2562 eve, weekends.
- SHARP MZ-80K; 48k. With hi-res graphics, worth over £100. Also manuals and many games (more than £50). Very little use. £225 ono. 0625 875039. Poynton, Stockport, Cheshire.
- BBC B. Dual double sided 80 track drives; own PSU. Epson MX 80III printer. Wordwise, graphics ROM, disc Doctor, Watford DFS. Lots of books, software etc. £1,000. Tel: St Albans 0727 36016.
- APPLE IIe, 3 Disk IIs, 2 Controllers, Monitor III, extended 80 column card and 32k Microbuffer printer card for sale. Bought Nov 83, upgrading to Apricot. Offers 03596-669.
- TRS-80, Model I, 48k expansion interface, 3 double density disk drives, monitor and heaps of software including Visicalc. £450 ono. Tel: 01-997 7578.
- Calculators: Texas TI 59, magnetic card, programmable, hardly used, immaculate £90. Hewlett Packard HP-34C, constant memory, programmable, perfect condition £38. Both complete with accessories and manuals. Tel: Castle Rising 520.
- TRS80 Model I; 48k, green VDU, lower case, twin 80 track drives. Newdos 80, Scrips + various programs £650 ono. Tel: Bishops Waltham 3752 evenings only.
- ATARI software discs. Unwanted gifts. Text Wizard word processor £48. Atari world £28. Tel: 0454 774309.
- TRS-80 Model 1 16k Level 2. Including green screen monitor, cassette leads, excellent software, manual and programming books. All in very good condition. £180. Tel: Milton Keynes 78884.
- TRS-80 Model I, Level II. 16k + 32k expansion interface, monitor, new 80-track disk drive, LDos 5.14 operating system, software on disk and tape (including cassette recorder). Moving hence offers over £425 accepted. Tel: John, 01-947 8442.
- DRAGON 32; disk drive, disks, tape recorder, tapes, printer cable, tape lead, Dragon user until April 1985, Sprite, 100 magazines, joystick, 5 books; worth £650, sell for £450 ono. Tel: (0602) 224046.
- HEWLETT PACKARD HP-41C. + 1 memory module and Math Pac. 4 months old. Guarantee still effective, £150 ono. Tel: 01-318 4201.
- APPLE type system. Twin drives, hi-res green monitor, 64k RAM, with printer £689. Also various add on cards. Tel: Jones on 0435 830680 for further details.
- PROFESSIONAL business system running CPM 2.2, Diablo daisy wheel printer with software including word processing, accounts, database, spreadsheet only £1,850. Jones. Tel: 0435 830680.
- TRS-80 Model 1 48k. Including monitor, expansion interface, disk-drive, TRS DOS and LP VII printer. Plus all manuals, £450 ono. Tel: Portsmouth (0705) 664911 (daytime) contact Colin Reynolds.
- SHARP MZ-80k. Excellent software including Sharp's Pascal, editor/assembler etc. Plus manuals, books and newsletters. As new condition in original box. Offers

- around £220. Tel: (061) 973 7701.
- NEWBRAIN. Complete with compatible cassettes recorder, leads, handbook, beginners guide and tape; £145. Tel: (061) 480 3438, during office hours. Stockport, Cheshire.
- EPSON HX 20. With tape drive, 1 year old, used once a week, £250. 01-681 7356 Scroggs, Croydon area. 178 Croydon Road, Beddington, Surrey.
- SELECTRIC keyboard printer, ex 2741 with hardware/software interface for Z80 micro + P10, runs with Nascom + NAS-SYS. Tandy etc. Previously IBM maintained, excellent working order. Sensible offers to Ian, Ipswich (0473) 831 353.
- SPECTRUM 48k. Interface 1, microdrive, ZX printer, printer paper. Books, ROM Disassembly, MicroDrive book, Beginner's M-Code. Hardware manual. Some software, offers around £230. Tel: (0258) 52508 after 6pm or weekends.
- FOR SALE. Video Genie 48k, £140, 12in screen monitor green £50, or part exchange for 2 disc drives suitable for Tandy TRS 80 Model 3. Genie has many extras. Tel: Jim. 0452-731358.
- PET 3032; C2N, green screen, soundbox, Arrow, over £400 worth including Apsai.
- Hitchhikers, Dragon's Eye, PetChess, and original Colossal Cave Adventure. £285 or + cash for Apple and disks. Tel: 748 7365.
- GEMINI Multiboard system. CPU, IVC, keyboard, 32k CMOS RAM card, Nascom I/O card, PSU + 12 slot card frame, also Gemzap, Gemdebug and Gempen. £350 ono. Tel: Brighton (0273) 506569.
- OSBORNE I. Excellent condition CP/M, Wordstar, Mailmerge, Supercalc, Basic & CBasic, TV adapter, 10 diskettes, comprehensive manual. £795 ono. Tel: Upminster (04022) 23886.
- OSBORNE I. Single Density CP/M, MBasic, CBasic, Supercalc, Wordstar, vgc. For quick sale £600 ono. Tel: 0276 21874 after 7pm or weekends.
- SINCLAIR SPECTRUM. With printer and many games including Jetpac, Transam, 3D Deathchase and many others. 16k machine. £130 ono. Tel: Monday to Friday after 6pm. Ask for Nasir on (021) 772 0600.
- SHARP MZ-8A, 48k RAM. Integral green monitor and cassette. Software includes 2 Basics. Home Finance, many games and tapes. House move forces sale, £350 ono. Tel: (01) 654 4556 (Croydon).
- VIC20 and C2N cassette unit, manual, books, presentation programs etc. All in good condition £90. S.E. London. Tel: (01) 635 8095.
- WANTED: TRS80 Model 1, 48k. With at least one disk drive. For sale: Tandy Quickprinter II £30, collected. Seven £4 packs quickprint aluminised paper for £15 post free. 20 Pytchley Drive, Loughborough LE11 2RH.
- ACULAB IEEE to parallel-port decoder type Pet 2 (7-bit); Aculab parallel to RS232 interface type 735P. Offers invited. Tel: Felixstowe (0394) 275378.
- TRS80 Model 1, 48k. RS232 Lodos, Newsript, Telesoft, Tanelt, Prestel, adaptor. Green VDU £220, or with two disks £480. Dr. Taylor, 11 Ironbridge Path, Fordham, Cambs CB7 5LJ. Tel: Newmarket 720765.
- VIDEO GENIE, EG3003, 16k, RAM. Built-in cassette, soundbox. Loads of software, Edtasm, Mon 3, Asteroids, Djinn and more. £180 ono. Tel: David, Burntwood (nr Walsall), Tel: (05436) 3729 eves.
- SHARP MZ80A 48k, £110, software includes Basic, Forth, Pascal, Machine Code Loader, Home Finance, Chess, Fender, Space Invaders, £375, 31 Burnhope Road, Barmston, Washington, Tyne and Wear. Tel: 091 4160865.
- SHARP MZ80K; £60 only. Not used much and nothing wrong with it. Repeat £60, only. Critchley, 10 Woodside Avenue, Brighton, BN1 5NF.
- BBC Model B, Zenith green screen monitor, cassette, £60 software (including Lisp, White Knight) £420, ono. Tel: 0703 617214.
- OLIVETTI PRAXIS. 35. Electronic daisywheel typewriter, suitable for adaption to daisywheel printer. £175. Tel: Guildford 223109 (eve).
- OSBORNE double density disks

NUMBERS

Question C: Summary of results for Kaprekar numbers

... continued from page 180.

n = 1	1	9
n = 2	(1) 45	99 55
n = 3	(1) 297	999 in six seconds, simple program 703
n = 4	(1) 2223 2728 4950	9999 total run time 0.97secs 7777 7272 5050
n = 5	(1) (4879) 17344 22222	99999 total run time 3.2secs 95121 82656 77778
n = 6	(1) (5292) (38962) 142857 148149 181819 187110 208495 318682 329967 351352 356643 390313 461539 466630 499500	999999 total run time 4min 10secs 994708 961038 857143 851851 818181 812890 791505 681318 670033 648648 643357 609687 538461 533170 500500
n = 7	(1) (627615) 4444444 4927941	9999999 total run time 39.5secs 9372385 5555556 5072059
n = 8	(1) (5479453) (8161912) 11111112 13641364 16590564 19273023 19773073 24752475 25252525 30884184 36363636 38883889 44363341 44525548 49995000	99999999 total run time 2 hours 3mins 94520547 91838088 88888888 86358636 83409436 80726977 80226927 75247525 74747475 69115816 63636364 61116111 55636659 55474452 50005000
n = 9	(1) 23467901 332999667 432432432	999999999 total run time 1min 6.7secs 765432099 667000333 567567568

Final comments

There are some significant intriguing patterns of digits and symmetries. In particular, the fact that the total of K-numbers for any n seems to be either 2, 48, or 32 indicates that it is possible to generate all the numbers by a smaller exploration still. There is a strong temptation to pursue the problem further . . .