NUMBERS

Factorials & primorials

Mike Mudge explores factorials and primorials which are near to prime.

Definitions

(i) A *prime number* is a positive integer which is divisible only by itself and unity. Thus the infinite sequence of prime numbers begins 2,3,5,7,11,13 . . .

(ii) The *factorial* of a positive integer, n, written n!, is defined to be the product of the positive integers less than or equal to n. Thus 6! = 1X2X3X4X5X6 = 720.

The sequence of factorials begins 1,2,6,24,120,720 . . .

(iii) The *primorial* of a prime number, p, written p*, is defined to be the product of the prime numbers less than or equal to p. Thus 7* = 2X3X5X7 = 210.

The sequence of primorials begins 2,6,30,210,2310 . . .

(iv) An integer, q, is said to be near-to-prime (NTP), if, and only if, either q+1 *or* q−1 are prime. (Note that if both q+1 and q−1 are prime then q is the mean of a prime pair; see Brun's Constant *PCW*, July).

Elementary Facts.

Factorial n is NTP for n=1,2,3,4,6,7 . . . since 2,3,(5,7), 23,719,5039 . . . are prime.

Primorial p is NTP for p=2,3,5,7,11 . . . since 3,(5,7), (29,31), 211,(2309, 2311) . . . are prime.

At least the first twenty-nine NTP factorials and the first seventeen NTP primorials are known; however, virtually nothing is known about their frequency of occurrence nor about their significance in analytic number theory.

Problem

Readers are invited to design and implement an algorithm for the determination of both NTP primorials and NTP factorials; attempting to reproduce and, if possible, extend the present results. Any possible suggestions as to the significance of these numbers would be most welcome.

Submissions should include program listings, hardware description, run time and output; they will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of £10 will be awarded to the 'best' entry received by 1 December 1984. Please address submissions to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF. Tel: (0902) 892141.

Review — Number Theories — March 1984

The original title was to have been Number Theory Nostalgia to emphasise the dates of the original solutions.

Submissions included the first Pro Pascal seen, from a Sirius 880 running at 5MHz; together with the expected Basic and Assembler programs running on NewBrain, Spectrum and BBC Model B computers.

(a) Complete solution *Math Quest Educ Times* vol 25 1876 p76. ($ax^5 \ldots dx^5$), $a+b+c+d=x, x-a=p^5 \ldots x-d=s^5, x=(1/3)(p^5+ \ldots +s^5)$ where p=3m, q=3m+1, r= 3m+2, s= 3m+3.

(b) *Amer Math Monthly* vol 2. 1895 ppl28-9.

(c) *Amer Math Monthly* vol 5. 1890 p114 also vol 8. 1901 pp48-9. Consider the solution 3^5-D, 3^5, and 3^5+D . . .

(d) *L'intermédiaire des math* vol 11, 1904, pp16-7; the only known exception is 23.

(e) *L'intermédiaire des math* vol 24, 1917 pp23-41; the only known addition being 8191 = $1+2+ \ldots +2^{12} = 1+90+90^2$.

This month's winner is John B Cook of 34 Joan Crescent, East Burwood, 3151 (232-2126), Australia, who used a TRS PC-2 with printer, as necessary.

John used 6.71 hours CPU time on (e) while Teilhet's limit of 600 in (d) was extended to 1800 in about 1½ days.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. END

LEISURE LINES

by J J Clessa

Quickie

Divide 10 pounds of sugar into three portions so that three times the smallest portion equals the middle portion, and four times the middle portion equals the largest portion.

Prize Puzzle

A test of logic this month.

On the island of Nonesuch there are five species of birds:

— the Auk
— the Bluebird
— the Cockatoo
— the Drongo, and
— the Egret

Four birdwatchers, Peter, Quentin, Roger and Stanley, are located at different parts of the island when five different birds fly over in rapid succession. Each man makes his own identification of the birds, and the results are:

	Peter	Quentin	Roger	Stanley
Bird 1	Bluebird	Egret	Cockatoo	Egret
Bird 2	Auk	Auk	Bluebird	Auk
Bird 3	Cockatoo	Bluebird	Drongo	Cockatoo
Bird 4	Egret	Bluebird	Egret	Bluebird
Bird 5	Bluebird	Drongo	Auk	Drongo

In fact, none of the birdwatchers identified all the birds correctly, but conversely, no one had them all incorrect either. No two birdwatchers had the same numbers of incorrect guesses, and each of the five birds was correctly identified by at least one birdwatcher.

What were the five birds?

Answers, on postcards only, to *PCW* Prize Puzzle September 1984, Leisure Lines, 62 Oxford Street, London W1, to arrive not later than last post on 30 September 1984

June Prize Puzzle

A massive response to the June puzzle — almost 400 entries were received, most of them with the correct solution. The problem was easily solved by micro by testing all possible 6-digit square numbers for the required conditions. The required numbers (excluding solutions with leading zeros) are:

$494209=(703)^2$

and

$998001=(999)^2$

The winning entry drawn at random from the pile came from Dr David Vaux of the John Radcliffe hospital, Oxford. Congratulations Dr Vaux. Your prize is on its way. END