NUMBERS

Binomial coefficients

by Mike Mudge

The Binomial Coefficients, variously denoted by ^nC_r or $\binom{n}{r}$ are defined by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ where integers n and r satisfy $0 \leq r \leq n$; and $r! = 1.2.3.\ldots.r, r>0$; $0! = 1$. For example. $\binom{20}{10} = 184756$.

These coefficients, whose common occurrences include the algebraic result $(A+B)^n = \sum_{r=0}^{n} \binom{n}{r} A^r B^{n-r}$ together with The Bernoulli Distribution in applied statistics are directly available (for sufficiently small n and r) on many scientific calculators.

However, since $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ the Binomial Coefficients may be written down using Pascal's Triangle. Thus:

n=1				1		1			
n=2			1		2		1		
n=3		1		3		3		1	
n=4	1		4		6		4		1

....

The array is edged with ones, while each interior term is the sum of the two terms immediately above it; r is counted along the diagonals of the array.

Let's turn our attention to certain results concerning the number theoretic properties of the $\binom{n}{r}$ as distinct from their usage in other branches of mathematics.

(1) If the binomial coefficients are factorised thus: $\binom{n}{k} = UV$ where every factor of U is less than or equal to k while every factor of V is greater than k there are known to be finitely many coefficients with $n \geq 2k$ for which $U>V$. Determine such cases for $k = 3,5,7,\ldots$. *Note.* If k=3 then n= 8,9,10,18,82 and 162 while for k=5, n= 10, 12, and 28 are known to be the only cases for $n \leq 551$.

(2) If n is a prime number greater than 3, the Wolstenholmes' Theorem states that $\binom{2n-1}{n}$ is congruent to 1 modulo n^3:

that is $\binom{2n-1}{n} = An^3 + 1$, where A is an integer. Display empirical evidence for this theorem and consider the possible truth of its converse.

(3) What can be said about the largest divisor of $\binom{n}{k}$ which is less than n?

If $n \geq k^2 - 1$ there is a prime divisor less than or equal to n/k apart from $\binom{62}{6} = 61474519$.

If $n<k(k+3)$ there is a prime divisor less than or equal to k + 1 apart from the cases $\binom{7}{3}$, $\binom{14}{9}$, $\binom{23}{5}$, $\binom{44}{8}$, and $\binom{47}{11}$.

How do we best generate such divisors?

(4) When is$\left(\binom{2n}{n}, 105\right) = 1$? That is, when do $\binom{2n}{n}$, and 105 have no common factor. RL Graham offered a prize of \$100 for proving that this happened infinitely often. At least fourteen values of n are known n = 1,10,756,...

If g(n) is the smallest prime factor of $\binom{2n}{n}$, then g(3160) = 13 and $g(n) \leq 11$ for $3160 < n < 10^{110}$. Does this help?

Readers are invited to investigate the above problems and results with an objective of extending empirical evidence and possibly generating new conjectures.

Submissions should include program listings, hardware description, run times and output. These will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of £10 will be awarded to the 'best' entry received by 1 January 1985.

Please address submissions to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs.

Review—Repunits—April 1984

Repunits have found favour far and wide: a most elegant work, conjecturing the primality of $aR_k + b$ when

a	b	k
3	−2	50&60
5	−2	66
7	2	66

and establishing the primality of $6R_{66} + 1$ in less than three hours in Forth on a Jupiter Ace arrived from Denmark.

A Sharp PC3201 using Z80 machine code on The Isle of Mull suggests that a prime p greater than 3 will be a factor of $R_{n(p-1)}$ for n=2,3,...

Substantial references on the subject of repunits are to be found in the paperback *Repunits and Repetends* by Samuel Yates published by The Star Publishing Company, Boynton Beach Florida 1982.

This month's winner is Robin Merson of 2 Vine Close, Wrecclesham, Farnham, Surrey, GU10 4TE. Robin appeals to any reader who has implemented the Primality Testing Algorithm described by H Cohen & HW Lenstra (*Mathematics of Computation,* vol.42. Jan 1984) to contact him on Frensham 3587 for an 'info' exchange with Hardy and Wright-type algorithms.

A summary of Robin's results follows. Further details are available from him or myself.

Using an Apple microcomputer several primes have been found of the form d(n)e, where d=5,6,7 or 8 and e has one or two digits. An embedded (n) in a number implies the previous digit is repeated n times. For example, 10(4)2 stands for 100002. The largest for each of the values of d are 5(92)21, 6(120)1, 7(99)1 and 8(138)1.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. END

LEISURE LINES

by J J Clessa

Quickie

Here is a remark attributed to that famous geometrician, PI Thagorus: 'Now I have a rough predictor of circle areas and volume.'

Prize Puzzle

And now one for the micros.

In the annual festival games held at the village of Little Dingblat in West Sussex, the main event is the marathon.

This year, all the entrants were numbered sequentially (1, 2, 3 . . .).

By coincidence, all those who completed the race carried numbers which were either exact primes or exact powers of other numbers. Furthermore, the total of the race numbers of the finishers was exactly equal to the total of the race numbers of those who dropped out.

How many entrants were in the race?

Answers, on postcards only, to Leisure Lines, *PCW* Prize Puzzle, October 1984, 62 Oxford Street, London W1, to arrive not later than 31 October 1984.

July Prize Puzzle

The problem was quite easily cracked by micro and several people sent in their programs and printouts.

The winning entry came from Adam Jefferson of Bradford, Yorks. Congratulations, Adam, your prize is on its way.

The solution to the problem was that the Dawsons and the Firths were the R C families—the children of these families were given £31.25 each.

Keep puzzling.