Triperfect numbers

Mathematical mind-benders from Mike Mudge

A positive number N is called a 'triperfect' number if and only if s(N)=3N, where s(N) denotes the sum of the positive divisors of N.

For example: $T_1=2^3.3.5=120$ is triperfect because 1+2+3+5+4+6+10 +15+8+12+20+30+24+40+60+120 =3.120 =360.

And: $T_2=2^5.3.7=672$ is triperfect because 1+2+3+7+4+6+14+21+8+ 12+28+42+16+24+56+84+168+112 +48+32+336+96+224+672=3.672= 2016.

We need not search for odd triperfect numbers since it has been shown that such numbers (if they exist) are:
(i) greater than 10^{50}, WE Beck & RM Najar, *Math Comp, vol 38, 1982, pp249-251.*
(ii) mutiples of at least 10 distinct prime factors, M Kishore *Math Comp, vol 42, 1984, pp231-233.*

However, six even triperfect numbers are known (including T_1 & T_2 above)

Readers are invited to design and implement an algorithm for the determination of some or all of these numbers.

Submissions should include program listings, hardware description, run times and output; these will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of £10 will be awarded to the 'best' entry received by 1 February 1985.

Please address entries to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF. Tel: (0902) 892141.

Diophantine equations

The response to this three-part problem displayed the considerable interest in the solution of equations in integers, both by computational and algebraic methods.

The wording of (1) was unfortunately somewhat misleading and prompted numerous telephone calls and letters; copies of the Barnes paper are available on request so that the detailed nature of the 'trivial' solutions may be seen.

The fourth non-trivial solution to (2) x=74, y=260 together with a further solution x=767, y=8672 hitherto unknown to the writer have been found. Are there any others?

In (3) the value x=3344161 was incorrectly printed but this did not deter extensive algebraic and computational analysis.

The May prize-winner is AS Tickner of 14 Grimsdyke Road, Pinner, Middlesex HA5 4PH.

The criteria used to justify the award for work on an HP-98020 A calculator are those listed together with a commendable tenacity of purpose. It is hoped that other contributors and those interested in Diophantine problems will contact Mr Tickner in an attempt to complete the analysis of problem (2) and to investigate further unsolved problems in this area.

Quotients of Wilson and Fermat

This proved to be a very popular competition, probably due to the simplicity of its formulation. A ZX81 ran for 32 days, a U2200 (Apple-compatible) in Pascal used 36-digit LONGINT to display explicit values of some F_p and W_p, while an Apple II in 14 days 3 hours and 22 minutes made a major contribution. However, several programmers were in difficulty with unrecognised integer overflow leading to incorrect solutions of $w_p=0(p)$.

Nonetheless, we can now list all the solutions of $a^{p-1}\equiv1 \pmod{p^2}$ for $2\leq a\leq31$ and $3\leq p\leq1040069$.

a	p
2	1093,3511
3	11,1006003
5	20771,40487
6	66161,534851
7	5,491531
10	3,487
11	71
12	2693,123653
13	863
14	29,353
15	29131
17	3,46021,48947
18	5,7,37,331,33923
19	3,7,13,43,137
20	281,46457
22	13,673
23	13
24	5,25633
26	3,5,71
28	3,19,23
30	7,160541
31	7,79,6451

(i) No solutions were found for a=21 or a=29.
(ii) There are no further solutions for a=2 with p less than 3×10^7.

Considering $(p-1)!=-1 \pmod{p^2}$. EH Pearson, *Mathematics of Computation, vol 17, 1963, p194* has tested all p less than or equal to 200183 and only p=5,13 and 563 satisfy the congruence . . .

The June prize-winner is Don Hunter of The Old Vicarage, Elmdon, Nr Saffron Walden, Essex CB11 4LT; whose ALGOL 60 programming of a 16k Elliott 903 using library procedures for multi-precision arithmetic ran for a total of 53½ hours.

The last such computer that the writer heard of had been refurbished following its removal from a tank.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. END

LEISURE LINES

Brain-teasers courtesy of J J Clessa

Quickie

What temperature is the same in Centigrade (Celsius) as it is in Fahrenheit?

Prize Puzzle

A certain 7-digit number contains no zeros and is not palindromic (that is, it does not read the same from right to left), but it does have the property that if its digits are reversed, the resulting 7-digit number is a factor of the original number. What is the original number?

Answers, on postcards only, to: *PCW* Prize Puzzle, November 1984, Leisure Lines, 62 Oxford Street, London, W1. Entries to arrive not later than 30 November 1984. END