NUMBERS

Mathematical mind-benders from Mike Mudge

This month sees a dramatic change in the area of research covered in Numbers, a move from integer arithmetic to floating point arithmetic.

Consider the decimal expansion of a fraction (which for convenience will be supposed to lie between 0 and 1). This either terminates, for example, 73/200 = .365 or yields a repeating pattern called a recurring decimal, say, 7/13 = .538461 538461 538461 . . . written .538461. There is little of interest for us in such cases.

However, suppose that we start with an irrational number, which by definition cannot be exactly represented as a fraction. What happens in its decimal expansion?

A number is said to be *simply normal* if each of the digits 0, 1, . . .9 occur equally often in the non-terminating expansion as a decimal; furthermore, it is said to be *normal* if every combination of these digits occurs with the proper frequency — by which we mean the frequency calculated on the assumption of randomness . . . the absence of any pattern.

A number which preserves the property of normality in every possible number system, including of course binary, is said to be *absolutely normal.*

Now we shall restrict our discussion to two famous irrational numbers:
(1) 'Pi', π, the ratio of the circumference of a circle to its diameter.

Readers may wish to use the series:
$\pi = 1/1\ (16/5 - 4/239) - 1/3(16/5^3 - 4/239^3) + 1/5(16/5^5 - 4/239^5) \ldots$ approximately 3.1415926536.
(2) 'e', the base of natural logarithms, defined by the series:
e = 1 + 1/1 + 1/(1.2) + 1/(1.2.3) + 1/(1.2.3.4) + 1/(1.2.3.4.5) . . . approximately 2.7182818285.

Readers are encouraged to examine and improve upon the very crude ZX81 Basic program given here.

```
 1 DIM X(100)
 2 DIM Y (10)
 3 DIM Z (175)
 4 DIM W (60)
 5 FAST
10 LET N = 40
15 PRINT "THE CALCULATION OF E
   TO ";N," DECIMAL PLACES
   YIELDS . . ."
20 LET M = 4
22 FOR D = 1 TO 10
23 LET Y (D) = 0
24 NEXT D
30 LET TE = (N + 1)* 2.30258509
33 LET M = M + 1
34 LET DI = M* (LN (M) - 1) + 0.5 ×
   LN(6.2831852* M)
35 IF DI <= TE THEN GOTO 33
36 PRINT " AN M REQUEST OF ";M
38 FOR J = 2 TO M
40 LET X (J) = 1
42 NEXT J
44 LET FI = 2
45 LET Y(2) = 1
48 FOR I = 1 TO N
50 LET CA = 0
52 LET J = H
54 LET TE = X(J)* 10 + CA
56 LET CA = INT (TE/J)
58 LET X(J) = TE - CA* J
60 LET J = J - 1
62 IF J >= 2 THEN GOTO 54
64 IF CA - 0 THEN GOTO 68
66 GOTO 70
68 LET Y(10) = Y(10) + 1
69 GOTO 72
70 LET Y(CA) = Y(CA) + 1
72 LET Z(I) = CA
74 NEXT I
76 PRINT FI; ".";
78 FOR Q = 1 TO 60
80 PRINT Z(Q);
81 NEXT Q
82 LET J = 61
84 IF J >= N THEN GOTO 97
86 LET M=J-1
88 FOR D = 1 TO 60
89 LET M = M + 1
90 LET W(D) = Z(M)
91 NEXT D
92 FOR E = 1TO 60
93 PRINT W(E);
94 NEXT E
95 LET J = J + 60
96 GOTO 84
97 PRINT
98 PRINT "END"
```

With each of the numbers π and e (and if a further challenge is needed the square root and cube root of 2), readers are invited to submit programs to calculate any required number of decimal places and to test at least for simple normality by counting the numbers of each digit present in the resulting decimal expansion.

Test Data: computed on ENIAC around 1950 took approximately 11 hours for e with a further 17 hours for card-handling and checking, and a total of around 70 hours machine running time for π. Readers will appreciate how computing has changed over the past quarter of a century (see Fig 1).

Submissions should include program listings, hardware description, run times and output; these will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of not less than £10 will be awarded to the 'best' entry received by 1 March 1985.

Please address entries to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs. WV4 5NF. Tel: (0902) 891141.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. Expanded reviews of previous 'Numbers' problems together with, subject to the approval of the contributor, copies of detailed programs from the prize-winning submission may also be requested. END

Digit	0	1	2	3	4	5	6	7	8	9
2,035 decimals of π	184	213	210	190	198	211	204	200	207	218
2,000 decimals of e	196	190	208	202	201	197	204	198	202	202

Fig 1

LEISURE LINES

Brain-teasers courtesy of J J Clessa

Quickie

Their are three mistakes in this sentense — can you find them?

Prize Puzzle

This one should keep the micros humming over the Christmas period. I believe the puzzle was originated by Ernest Dudeney but it's certainly been around a while.

Can you find an integer which — divided by 5 and multiplied by 4 — gives the same result if you move the first digit of the number to the end.

For example, suppose the number is 2615. If you divide it by 5 and multiply it by 4 you get 2092. But if it were the number we were seeking, we would get 6152.

Answers please, on postcards only, to: *PCW* Prize Puzzle, December 1984, Leisure Lines, 62 Oxford Street, London W1. Entries to arrive not later than 31 December 1984.

August Prize Puzzle

The answers are as follows:

(a) The largest perfect square with digits in ascending order is 134 689.

(b) The largest perfect square with digits in descending order is 961.

Winner: D Haworth, Bolton, Lancs. Congratulations! END

Dec 84