south-west. This group contained the same number of birds as were heading south-east. There were now three exact

definition of a prime is an integral number which has no factors other than unity and itself'. We would be in-

son of Wisbech, Cambridgeshire. Congratulations, John.

NUMBERS COUNT

Mike Mudge delves into Euler's Totient function and presents further solutions to the Collatz problem.

Euler's Totient function

The great mathematician Leonhard Euler (1707-1789) had his dormant interest in number theory awakened by certain results of Pierre de Fermat (1601-1665). From 1747 to his death, the last thirteen years suffering total blindness, he made many valuable contributions in the field of number theory.

Theory and Definitions Euler's Totient function $\phi(n)$, is defined to be the number of numbers not greater than n and prime to n. (That is, the number of numbers less than or equal to n and sharing no factor with n.)

n	1	2	3	4	5	6	7	8	9	10	11......	50
$\phi(n)$	1	1	2	2	4	2	6	4	6	4	10......	20

Nontotients are those positive values of n for which $\phi(x)=n$ has no solution for example: 14,26,34,38.

Noncototients are those positive values of n for which $x-\phi(x)=n$ has no solution for example: 10,26,34,50.

Now we define $f(n)=n-\phi(n)$ and observe that f(n) is less than n; thus if we iterate the function f to obtain f(f(...f(n)...)) we *must* eventually reach 1. For example: $f(6)=6-2=4$, $f(4)=4-2=2$, $f(2)=2-1=1$. Write s(k) to be the smallest integer which reaches 1 after k iterations.

k	2	3	4	5	6	7
s(k)	4	6	10	18	30	42
	2.2	2.3	2.5	2.3.3	2.3.5	2.3.7

Question 1 Is there a pattern to the factorisation of s(k)? Is s(8)=2.3.5.7 or 2.3.3.5 or 2.3.11?

Question 2 (a) Are there infinitely many pairs of consecutive numbers n and n+1 such that $\phi(n)=\phi(n+1)$? For example: n=1,3,15,104.

Note that 18 solutions are known less than 10^4 and 59 less than 10^6.

(b) What about solutions of $\phi(n)=\phi(n+1)=\phi(n+2)$? (c) Consider $\phi(n)=\phi(n+2)=\phi(n+4)$ (d) Similarly $\phi(n)=\phi(n+3)=\phi(n+6)$ and so on.

Note that Schinzel has conjectured (1958) that $\phi(n+k)=\phi(n)$ has an infinity of solutions for every k. However for k=3 only the solutions n=3 and n=5 are known.

Question 3 Determine the number N(y) of nontotients less than y as a function of y, extending the following table.

Y	10^3	10^4
N(y)	210	2627

Question 4 How many noncototients are there less than a given y?

Question 5 Is there a non-prime integer n, such that $\phi(n)$ is a divisor of n-1?

Readers are invited to submit their program listings, output and hardware details together with their conclusions relating to some or all of the above questions to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffordshire, WV4 5NF. Tel 0902-892141.

A suitable prize will be awarded to the best entry received by 1 August 1985. Criteria will include accuracy, originality and efficiency, not necessarily in that order.

Please note that submissions can only be returned if a suitable stamped addressed envelope is included. Expanded reviews of previous problems together with, subject to the approval of the contributor, copies of detailed programs from the prize winning entry may also be requested.

Prize winner November

Numerous investigators discovered the six triperfect numbers mentioned; $T = 2^3.3.5 = 120$, $T_2 = 2^5.3.7 = 672$, $T_3 = 2^9.3.11.31 = 523776$, $T_4 = 2^8.5.7.19.37.73 = 459818240$, $T_5 = 2^{13}.3.11.43.127 = 1476304896$ and $T_6 = 2^{14}.5.7.19.31.151 = 51001180160$.

A very careful analysis by H Ibstedt of Paris in Basic on an ABC80 Metric with 32 kbyte RAM manufactured in 1979 by the Luxor company obtained these results in 3 minutes 45 seconds computing time.

However this month's prizewinner has to be Mr RF Tindall from Cambridge who has, to the best of my knowledge, pushed back the frontiers of knowledge with T_7 which is $2^{24}.3.19^2.31.113.127.151.301.451.601.901.1801$ the remarkable aspect of this work is that *no* computer has been used, Mr Tindall has also investigated 4,5,6 and 7-fold perfect numbers; he refers to AH Beiler, *Recreations in the theory of numbers* '. . . as having many useful tables', and I feel sure would be pleased to discuss his algorithm (of which I also have the details) with any aspiring programmers who would like their computer to deduce T_7.

Collatz problem

Recall the iterative scheme $x_{n+1} = x_n/2$ if

NUMBERS COUNT

x_n is even else it is $3x_n + 1$; x_0 arbitrary. Does the scheme always reach 1? Mr. Jacoby Thwaites from London has constructed a truly remarkable program for the BBC Micro in Basic crossed with 6502 assembler which inputs an arbitrary x_0 in hexadecimal less than or equal to about 10^{40000} and applies the above iterative scheme displaying the x_n in binary as the iteration proceeds and counting the steps for example: F(1000), the hexadecimal number represented by F repeated 1000 times undergoes 19794 increases and 15579 decreases before reaching unity. (Run time a few minutes).

This represents a totally new area of work for such a micro; I would like to congratulate Mr Jacoby on this achievement and encourage all readers interested in the approach to contact him, (01) 242 9210.

MICROCHESS

Kevin O'Connell comments on a match between International Master, Julian Hodgson and Conchess.

England now has a very strong claim to rank as the second chess nation on earth, behind only the Soviet Union. Julian Hodgson, an International Mas-

Qh4xe4+ 6 Ke2-f2 Bf8-c5+ 7 Kf2-g3 Qe4xe5+ I would really enjoy playing Black's position against a computer opponent.)

15	...	Bf8-e7
16	Ng3-e4	f7-f5
17	Nd5xe7	Qd8xe7
18	Ne4xd6+	Ke8-f8

(Now White, two pawns up and with