whose product is 11 (for the uninitiated, arithmetical progression means that

March Prize Puzzle

What a load of ball-bearings! There was

tulations, your prize is on its way

NUMBERS COUNT

Mike Mudge offers a clue to the mystery of the Riemann Hypothesis, and presents the winning solution to the theory of Normal Numbers.

The Moebius Function

Definition. The function of Moebius $\mu(a)$ is defined for all positive integers a by the equalities $\mu(a) = 0$, if a has a squared factor distinct from 1, $\mu(a) = (-1)^k$, where k denotes the number of prime divisors of a, and a greater than 1 has no squared factor distinct from 1. In particular, for a = 1 we assume that k = 0 and therefore take $\mu(1) = 1$.

a	1	2	3	4	5	6	7	8	9	10	11	12
$\mu(a)$	1	−1	−1	0	−1	1	−1	0	0	1	−1	0

Check: the sum of the values of $\mu(a)$ from a = 1 to 100 is 1.

Alternative definition: $\mu(1) = 1$, $\mu(p) = -1$, $\mu(p^n) = 0$ for n greater than 1, $\mu(mn) = \mu(m)\,\mu(n)$ when m and n are coprime; that is, m and n have no common factor other than 1. p denotes a prime.

This function is thus very easy to evaluate for isolated numbers whose factors are known, and attention has focused on its use to evaluate more complicated allied functions.

In 1884 J P Gram published $\mu(n)$ together with the sum:

$$S_n = \sum_{k=1}^{n} \mu(k)k^{-1}$$

for $n \leqslant 300$; subsequently Euler's conjecture that as n tended to infinity S_n tended to zero was rigorously proved.

The function:

$$M_n = \sum_{k=1}^{n} \mu(k)$$

has, however, attracted a great deal of attention. In 1897, F Mertens tabulated $\mu(n)$ and M_n for $n \leqslant 10000$; over the period 1897 to 1912, RD von Sternbeck tabulated M_n for all $n \leqslant 150000$, then in steps of 50 up to 500000, followed by 16 values chosen in the range from 600000 to 5000000. These tables were constructed with the hope of shedding some light on the problem, believed to be still unsolved, of the behaviour of M_n for sufficiently large n. This problem is intimately connected with the Riemann hypothesis, whose consequences are to be found throughout classical number theory.

The Moebius function is related to Euler's Totient Function (see *PCW*, May) thus: if $\phi(n)$ denotes Euler's Totient Function and

$$\Phi(n) = \sum_{v=1}^{n} \phi(v),$$

where it is known that $\Phi(n) = 3n^2/\pi^2 + o(n \log n)$ (order of n log n), then the sum of $\phi(v)$ is most easily calculated from the formula in Fig 1 where [x] denotes the largest integer not greater than x and M(x) denotes

$$\sum_{v \leqslant x} \mu(v)$$

that is, the generalisation of M_n in the obvious way. JWL Glaisher (1940) published tables of $\Phi(n)$ for n going from 1000 to 10000. It is interesting to note that $E(n) = \Phi(n) - 3n^2/\pi^2$ is positive for all $n \leqslant 1000$ except n = 820.

There are further topics, such as the theory of irreducible cyclotomic polynomials involving the Moebius function, but these are beyond the scope of this article.

Problem Tabulate the functions $\mu(n)$, S_n, M_n and possibly $\Phi(n)$, the latter to be calculated using the above formula.

Demonstrate the plausibility of the now-proven Euler conjecture referred to above, speculate on the behaviour of M_n, demonstrate the anomalous sign of E(n) at n = 820, and attempt an explanation.

Readers are invited to submit their program listings, output and hardware details together with their conclusions relating to this problem to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF. (Tel: (0902) 892141). A suitable prize will be awarded to the 'best' entry received by the 1 September 1985. Criteria will include accuracy, originality and efficiency.

Please note that submissions can only be returned if a suitable stamped addressed envelope is included. Expanded reviews of previous problems, together with, subject to the approval of the contributor, copies of detailed programs from the prize winning entry may also be requested.

$$2\Phi(n) - 1 = \sum_{v=1}^{[\sqrt{n}]} \left\{ \left[\frac{n}{v}\right]^2 \mu(v) + M\left[\frac{n}{v}\right](2v-1) \right\} - M(\sqrt{n})[\sqrt{n}]^2$$

Fig 1

NUMBERS COUNT

Prize-winner December

This topic produced a most interesting and varied response, including Tansoft Forth on an Oric Atmos and 'Homebrew' Forth on an Image 8.

A detailed study of the m^{th} root of n in Z80 code, using the Zeus Assembler on a Spectrum, was particularly interesting. Fortran 77 on a CDC Cyber 180-810 appeared from Sweden, and Z80 code using a two-pass compiler on a ZX81 came from West Germany. A Spectrum, fitted with a 7608 voltage limiter, ran in Basic for 100 hours to duplicate the ENIAC result for pi.

This month's winner is Ronald B Shepherd of Cottingham, Humberside, who used Prospero ProPascal version zz 2.1 on a Sharp MZ80B (64k, Z80A, clock frequency 4MHz) with twin 5¼in floppy disks and printer. Ronald calculated e to 5000 digits (2000 in 54 minutes) and pi to 2000 digits (6¼ hours), and having written these to disk analysed them statistically. This analysis, including frequency, runs and serial test, was based upon the algorithms of WJ Kennedy and JE Gentle, *Statistical Computing*, 1980. The whole work was extremely well-documented with references, full listings and tabulated output. Suggestions for further work included the gap test and a study of the m^{th} root of n using Newton's Method, together with the digital analysis of Euler's Constant, gamma.

A well-deserved prize is on its way to Humberside.

MICROCHESS