END ZONE

MICROCHESS

33	Qd2–g5+	Kg7–f8	
34	Qg5–h6+	Kf8–e8	
35	Bc2–g6+	Ke8–d8	
36	Qh6–h8+	Black resigns	

(It is mate in two moves: 36. . .Be7 –f8 37 Qh8xf8+ Nf6–e8 38 Qf8xe8.)

NUMBERS COUNT

Mike Mudge investigates problems in the theory of continued fractions.

Definition An expression of the form:

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{\ldots}}}}$$

(where $a_0, a_1, a_2, a_3, \ldots$ are positive integers) is called a regular (or simple) continued fraction.

For ease of printing it is written $(a_0;a_1,a_2,a_3,\ldots)$, the a_i are called the partial quotients.

Theorem I Given any rational number greater than zero, that is a fraction p/q where p and q are positive integers with no common factor, the associated continued fraction is finite.

131/17 = (7;1,2,2,2) =

$$7 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}}$$

Theorem II (The converse of theorem I.) Any finite continued fraction represents a rational number. (Loosely we may say that any finite continued fraction can be 'wrapped up'!)

(2;3,1,7,) = 70/31

Theorem III (See for example *Continued Fractions* by A Ya Khinchin, translation from the Russian published by The University of Chicago Press in the Phoenix Science Series 1964, pp 47-50.)

Any positive real root of a quadratic equation has a periodic simple continued fraction.

$2\frac{1}{2} = (1;2,2,2,..) = (1;\dot{2})$

$3\frac{1}{2} = (1;1,2,1,2,1,2,\ldots) = (1;\dot{1},\dot{2})$

$7\frac{1}{2} = (2;1,1,1,4,1,1,1,4,1,1,1,4,\ldots) = (2;\dot{1},1,1,\dot{4})$

The converse is also true, any periodic continued fraction represents the root of a quadratic equation.

$x = (1;7,7,7,\ldots) = \left(1;7,\frac{1}{x-1}\right)$

here $x = 1 + \cfrac{1}{7 + (x-1)}$

thus $x^2 + 5x - 7 = 0$ and $x = (53^{1/2} + 5)/2$ approximately 1.140054944

It should be noted that (1;7) is approximately 1.142857142 (1;7,7) = 57/50 = 1.14 while (1;7,7,7) = 407/357 is approximately 1.140056022.

Certain special continued fractions are to be found in the literature, for example, *Continued Fractions* by C D Olds Appendix II includes (Euler 1737)

e = (2;1,2,1,1,4,1,1,6,1,...) = (2;1,2n,1) n = infinity n = 1

also (Lambert 1770)

Pi = (3;7,15,1,292,1,1,1,2,1,3,1,14,2,1,1, 2,2,2,2,1,84,2,...) with no apparent pattern.

However general results are somewhat sparse. In the summer of 1970 the Maniac computer at Los Alamos used 25000 decimal digit arithmetic to calculate the first 8000 partial quotients in the continued fraction expansion of the cube root of two; the theoretical interest centring around the statistical distribution of these a_i.

Nearer home, *J Inst Maths Applic* (1969) Vol 5 pp 318-328 R F Churchhouse and S T E Muir report that the real root of $x^3 - 8x - 10 = 0$ (approximately 3.318628217750185) was calculated to 200 decimal places and the first 200 partial quotients (beginning (3;3,7,4,2, 30,1,8,3,1,1,1,9,2,2,1,3,22986,2)) were also determined in a total of 10 seconds on Atlas at S R C Chilton!

Problems 1) Determine the period of the continued fraction expansion of a given quadratic irrational that is, $p + q(x)^{1/2}$. . try $(4517^{1/2} - 61)/3$.

2) Determine the continued fraction expansion of a real number given to an arbitrary precision. Try 0.1234 5678910111213141516.

3) Compute exactly the rational number (fraction) corresponding to a given finite continued fraction. Try (1;2,3,4,5,6,7,8,9,..n)

4) Compute the positive real root(s) of a given cubic (or higher degree equation) equation to arbitrary precision and use the result of (2) to find the continued fraction expansion.

The statistical distribution arising in this theory will be discussed in a later Numbers Count article if the response warrants it.

Readers are invited to submit their program listings, together with hardware descriptions, run times, any comments and of course the output relating to the above problems. These submissions will be judged, using suitably vague criteria, and a prize will be awarded to the 'Best' entry received at 'Square Acre', Stourbridge Road, Penn, Nr. Wolverhampton, Staffordshire WV4 5NF; (tel (0902) 892141) by 1 January 1986.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. Expanded reviews of previous problems together with, subject to the approval of the contributor, copies of detailed programs from the prize winning entry may also be requested.

```
MULGEN VER. 850621 TIME: 00:00:02

INPUT MULTIGRADE OF ORDER  6:
A:  39  38  28  27  16  10  5  1
B:  40  36  31  25  14  13  3  2
TRANSLATION =  150

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NUMBER OF TERMS = 8
A:  151  155  160  166  177  178  188  189
B:  152  153  163  164  175  181  186  190
POWER SUMS:
A ^ 1 =  1364                   B ^ 1 =  1364                   EQUAL SUMS FOR EXP. =  1
A ^ 2 =  234060                 B ^ 2 =  234060                 EQUAL SUMS FOR EXP. =  2
A ^ 3 =  40418048               B ^ 3 =  40418048               EQUAL SUMS FOR EXP. =  3
A ^ 4 =  7022349636             B ^ 4 =  7022349636             EQUAL SUMS FOR EXP. =  4
A ^ 5 =  1227303597344          B ^ 5 =  1227303597344          EQUAL SUMS FOR EXP. =  5
A ^ 6 =  215710267243380        B ^ 6 =  215710267243380        EQUAL SUMS FOR EXP. =  6
A ^ 7 =  3.811648835146717D+16  B ^ 7 =  3.811648824139357D+16
A ^ 8 =  6.769293083177564D+18  B ^ 8 =  6.769292933037173D+18
TIME: 00:01:14
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Fig 1

Prize winner April

The first response to Sums of Powers used a simple approach in Basic on a TI-99/4A, subsequent contributors ranged over the spectrum of combinations of theory and empirical programming and over the globe from Oxford to Cambridge to Saudi Arabia.

The winner this month is Henry Ibstedt of 4, rue Gramme; 75015 Paris, a regular contributor to the mail bag, with a very well presented combination of theorems with proofs, followed by detailed implementation in Basic for an IBM PC with 256 kbyte Ram. The detailed results would suffer badly from the condensation needed to fit the available space. Fig 1 gives a sample of Henry's style.