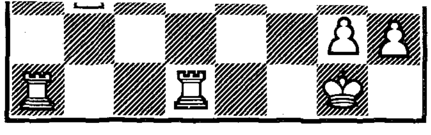

The computer's task could be harder

been better.)

30	...	d4-d3!
31	Rc1-c7	d3-d2!
32	Kg1-f1	

(32 Rc7xb7 loses horribly to 32...Re2-e1+.)

49	Kg3-f3	Rc2-h2

0-1

(White resigns because he cannot defend the h-pawn.)

NUMBERS COUNT

Mike Mudge presents prime period lengths and some related conjectures.

The period length L(p) of a prime number p is defined to be the number of digits in the period, or repetend, of the decimal representation of a proper fraction whose denominator is p.

Therefore, L(3) = 1 because 1/3 = .333 ... the period having one digit, namely 3. However, L(17) = 16 because 1/17 = .05882352941176470588235294117647 ... or as one learnt at school 1/17 = .0588235294117647, the period having 16 digits.

Simple consideration of the division process and the remainders which arise at each stage will convince the reader that L(p) is less than p, and if L(p) = p − 1 the p is called a *full period prime*, for example 17, 19, 23, 29.

Results

1) L(p) divides p − 1, so that p is one greater than a multiple of L(p).
2) Every positive integer is the period length of, at most, a finite number of primes.
3) Every positive integer is the period length of at least one prime.
4) The period lengths of all primes congruent to 7, 11, 17, 19, 21, 23, 29 or 33 (modulo 40) are even, and divisible by the largest power of two that divides p − 1.
5) The period lengths of all primes congruent to 3, 27, 31 or 39 (modulo 40) are odd.
6) If p and 2p + 1 are both prime, and if 2p + 1 is congruent to 7, 19 or 23 (modulo 40) then L(2p + 1) = 2p.
7) If p and 2p + 1 are both prime, and if 2p + 1 is congruent to 3, 27 or 39 (modulo 40) then L(2p + 1) = p. (Recall that a is said to be congruent to b modulo c if, and only if, the difference between a and b is a multiple of c.)

Asymptotic conjectures

(i) Three-eighths of all primes are full period primes.
(ii) The period lengths of all primes are distributed evenly among the 16 possible residue classes modulo 40.
(iii) The period lengths of half of all primes congruent to 13 or 37 modulo 40 are even, and half are odd.
(iv) The period lengths of five-sixths of all primes congruent to 1 or 9 modulo are even, and one-sixth are odd.
(v) The period lengths of two-thirds of all primes are even, and one-third are odd.
(vi) If the primes are divided into three categories, a) full period, b) odd period, c) non-full period with even period, the ratio of the totals in each is 9:8:7.

Many of these results are due to Samuel Yates of the US, and have been published by him in *The Journal of Recreational Mathematics* and elsewhere. His investigations have encouraged numerous other empirical number theorists to investigate prime periods and to advance results equivalent to those presented here.

Readers are invited to submit their program listings for the determination of prime periods, and hence the examination of the above results and conjectures. Particular interest centres around the generalisation of this work to number bases other than 10.

These submissions, which should include hardware description, run times, any comments, and of course some specimen output, will be judged using suitably vague criteria. A prize will be awarded, by *PCW*, to the 'best' entry received at 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffordshire WV4 5NF (tel: (0902) 892141) by 1 February 1986.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. Expanded reviews of previous problems together with, subject to the approval of the contributor, copies of detailed programs from the prize-winning entry may also be requested.

Prize-winner May

Some general introductory material relating to Euler's Totient function can be found, for example, in *A Pathway into Number Theory* by RP Burn, CUP 1982, or *An Introduction to the Theory of Numbers* by IM Vinogradov, transla-

END ZONE

NUMBERS COUNT

tion published through Pergamon Press 1955. Detailed references to numerical results are given in *A Handbook of Integer Sequences* by NJA Sloane, Academic Press 1973.

This month's prize-winner is Robin Merson of 2 Vine Close, Wrecclesham, Farnham, Surrey GU10 4TE (tel: Frensham 3587) whose achievements include the following:

(i) Solutions of phi(n) = phi(n + 1) from n = 8 up to n = 20171384.. run time 16 days.
(ii) All nontotients less than 10^4, with verification of 210 less than 10^3 and 2627 less than 10^4.
(iii) All noncototients less than 30000.
(iv) Determination of s(k) up to s(43) = 12531330
(v) Print of nontotients $2^k m$ with k greater than 3 and m a product of at least three different primes up to 547872.
(vi) Numbers in arithmetic progression and having the same phi, including six in a row: 165488430, 165488460, 165488490, 165488520, 165488550 and 165488580, all with phi equal to 44130240.

LEISURE LINES

Brain-teasers from JJ Clessa

Quickie

Another old chestnut. There are eight pints of milk in a churn. Using a three-pint jug and a five-pint jug only,

all digits 1-9.

As this can be done in more ways than one, you must find:

(a) The smallest 9-digit number; and
(b) The largest 9-digit number.

rect solutions to the problem of the number which, when multiplied by 13 and divided by 11, gives the original number in reverse.

Many of you pointed out that zero