NUMBERS COUNT

Mike Mudge looks at two-way numbers — a possible revolutionary approach to arithmetic?

This month's story begins with a paper by John Colson entitled *Negative-affirmative arithmetic*, published in *Philosophical Transactions of the Royal Society*, vol 34, pp161-174 in 1726. In this paper a new way of counting was proposed which used numbers −4, −3, −2, −1, 0, 1, 2, 3, 4, and 5. (No mention of 6, 7, 8 or 9 to be made). The idea was encouraged by the famous mathematician Augustin-Louis Cauchy in 1840, *Comptes Rendus*, vol 11 pp789-798, and formalised by *J Halcro Johnston* in his book *The Reverse Notation*, Blackie 1938.

Notation. The conventional notation consists of inverting the symbols 1, 2, 3 and 4 to represent −1, −2, −3 and −4. However, not having this facility available here, we shall use %, ½, ⅓, and ¼ respectively.

It is hoped that readers experimenting with two-way numbers on their computers may be able to print the conventional inverted digits in their output; the alternative is to use the *non-standard* notation of this article.

Continuing with the background to this concept, a number of papers by Cedric A B Smith from 1941 to 1974 attempted to expound the advantages of the two-way system and were supported by Dylan Morgan writing under the title *Arithmetic made easy, New Scientist*, April 1982; also by the publication as an appendix to *Biomathematics* vol 2, 1969 of *Two-way tables*, including squares, common logarithms, sines and cosines.

The quarterly journal *Colson News*, ISSN 0265-9905 first appeared in 1984 and has now produced seven editions (printed at University College, London and edited by CAB Smith, E Hillman and A Paul). It is appropriate to acknowledge the assistance which this publication has given to the preparation of this article.

Two-way numbers. The sequence of positive integers now appears thus: 1, 2, 3, 4, 5, 1¼, 1⅓, 1½, 1%, 10, 11, 12, 13, 14, 15, 2¼, 2⅓, 2½, 2% . . . where ¼ means −4, ⅓ means −3, ½ means −2, % means — For example, the year 1984 is thus written 20½4, 1985 becomes 20½5 while 1986 is written 20%¼.

It is appropriate to record here that J Halcro Johnston who can be aptly called 'The Father of Modern Two-Way Numbers' died on January 3¼, 20½4 at the age of 1%2, and it would be a fitting tribute if his concept of arithmetic could be implemented in a genuine manner on a digital computer.

Two-way arithmetic. We have the additional operation of inverting each of the digits, noting that 0 is self inverse, as is 5 in our presentation; although closer study of two-way numbers will reveal certain advantages in having available a symbol for −5, conventionally 5 inverted but in our notation it would have to be 1/5.

Subtraction then becomes inversion followed by addition; multiplication is somewhat easier in two-way arithmetic and division is little more than a series of additions. Thus:

1× ⅓¼ ÷ ⅓¼) 133234 (4%1% = 4%2%
2 × ⅓¼ = %32%¼4 3 × ⅓¼ = %0½ ⅓2
4 × ⅓¼ = %¼4 34 5 × ⅓¼ = ½30 1¼3
⅓¼ 54 34 %½

Compare with 133234/34 = 3918 remainder 22

Problem A. Construct a computer program which will convert both integers and real numbers expressed as decimals from one-way to two-way notation and vice-versa.

Check. Pi = 3.14159265 . . . = 3.142¼%3⅓1/5 . . .
√3 = 1.7320508 . . . = . . . = 2.⅓3211/51½ . . .

Problem B. Construct subroutines for carrying out genuine two-way arithmetic (not simply converting to one-way, carrying out the arithmetic and then reconverting). For example, 1½½11%^{3} = 5½⅓1/54425%45%03% *cf* 78109^3 = 476544249449029.

Problem C. Consider the decimal expansion of 1/n in two-way arithmetic, case:

(i) the expansion is finite,

(ii) the expansion is alternating — for example, 1/1⅓ = .143%¼⅓143% ¼⅓ . . ., 1/13 = 1½⅓%231½% 23 . . .; and

(iii) the expansion is recurring and non-alternating.

Use a computer program to classify 1/n under types (i), (ii) or (iii) for a large range of values of n. What can be concluded?

Readers are invited to submit the results of their investigations into the use of two-way numbers on a computer to: Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Wolverhampton, Staffordshire WV4 5NF. Tel: (0902) 892141.

Submissions, which must reach me by 1 April 1986, will be judged using suitably vague criteria. A prize will be awarded for the best entry received.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Expanded reviews of previous problems together with, subject to the approval of the contributor, copies of detailed programs from the winning entry may also be requested. However, interested readers are urged to contact the prizewinner directly.

The Numeri Idonei — July review

This problem brought responses from places as far apart as Australia and West Yorkshire. John Verey in Westmead, Australia used a TRS80 Model 1 Level 2 with 16k memory in Basic to test about 48,000 odd numbers.

This month's prizewinner, Gordon Mills of 6 Denehill, Bradford, West Yorkshire BD9 6AT who 'had some fun with this problem', used an Apple II Europlus with Applesoft Basic. He correctly modified the problem as posed by the addition of the condition that 'n' can also be a power of any prime or twice the power of any prime. His two-stage sieve process yielded all the Numeri Idonei and lead to the conclusion that the chance of finding a NI greater than 10000 is extremely small; although it cannot be ruled out.

'I can't help thinking there's something vaguely familiar about this new Sinclair.'