# MICROCHESS

played Fidelity Elite XC, the latter being the company's favourite. Avant Garde emerged from a scrappy game that rapidly simplified with a clear and easy win — a pawn up in a bishop ending. However, the operator of the Avant Garde suddenly decided to resign for his machine!

David Kittinger, programmer of the Novag machines, summed up many of the participants' reactions when he said of this: 'To me, resigning in a won position goes beyond chess, no matter whether it is in the rules or not. You cannot throw a game. This is fixing a game, it is absurd.'

Fidelity's president later justified the action by saying: 'It is not in the rules, therefore it is allowed.'

This, then, was a very serious omission, either from the rules or by the tournament referee. The most important events, such as the World Micro Championship, are governed by rules which were developed by David Levy and myself for the *PCW* computer chess events which were a regular feature of the *PCW* shows until 1984. We stated that the computer event rules were in addition to all the normal rules that apply in human competitions. The only comparable example I know of from human play resulted in both players being awarded a loss. Such things, of course, can affect the outcome of whole competitions.

# NUMBERS COUNT

*Mike Mudge delves into Diophantine Equations with Markoff Numbers.*

**Definition.** A 'Markoff Triple' is a triple of positive integers (p,q,r) which satisfies the equation: $p^2+q^2+r^2=3pqr$.

There are two singular Markoff Triples (1,1,1) and (2,1,1) and, since the equation is quadratic in each of the variables, it is known that one integer solution leads to another. It is further known that apart from the two singular solutions displayed above, all other solutions have distinct (different) values for p, q and r. Each solution is, therefore, said to be a 'neighbour' of just three others; these being the three with which it shares just two common values.

**Note.** A Markoff Triple is not ordered, since the equation is symmetrical in p, q and r then (p,q,r), (p,r,q), (q,r,p), (q,p,r), (r,p,q) and (r,q,p) are indistinguishable.

For example, (433,5,29) is a neighbour of (29,5,2), (6466,5,433) and of (37666,433,29) because:

$433^2 + 5^2 + 29^2 = 3.433.5.29 = 188355$:

$29^2 + 5^2 + 2^2 = 3.29.5.2 = 870$:

$6466^2 + 5^2 + 433^2 = 3.6466.5.433 = 41996670$:

$37666^2 + 433^2 + 29^2 = 3.37666.433.29 = 1418915886$:

and further each of the last three triples in the example shares two common values with the first.

The ordered list of numbers found in Markoff Triples defines the sequence of Markoff Numbers, of which there are only thirteen less than one thousand. Recall that the sequence of Fibonacci Numbers is defined algebraically, thus $F_1 = 1, F_2 = 1, F_n = F_{n-1}. + F_{n-2}$. Thus $(F_n)$ = (1,1,2,3,5,8,13,21,34 . . .).

It will be found that eight of the first 13 Markoff Numbers are also Fibonacci Numbers.

**Problem**

(ia) Determine all the Markoff Numbers less than any given integer; count how many of them are also Fibonacci Numbers.

(iib) Using the fact that each Markoff Triple has three neighbours, display these triples in the form of a tree and answer the outstanding question of whether or not the display is a genuine binary tree: can the same Markoff number be generated by two different routes through it (see Fig 1)?

(1,1,1)
↓
(2,1,1)
↓
(5,1,2)
↓ ↓
(13,1,5) (29,5,2)
↓ ↓ ↓ ↓
(34,1,13)(194,13,5)(433,5,29)(169,29,2)

*Fig 1*

A related Diophantine Equation is:

$5(p^2+q^2+r^2+s^2+t^2)^2 - 7(p^4+q^4+r^4+s^4+t^4) = 90pqrst$

Now apart from trivial solutions such as (1,1,1,1,1) and (2,1,1,1,1), the only solution which the author is aware of is (2,7,19,47,59) when each side of the equation equals 66385620, the individual terms on the left-hand side being 186294080 and 119908460.

**Problem**

(iia) Obtain the above solution by a search technique.

(iib) Extend the bounds up to which this is the only known non-trivial solution from p,q,r,s,t less than 100 and hopefully discover other solutions.

Readers are invited to submit their attempts at problems (i) and (ii) above to: Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Wolverhampton, Staffordshire WV4 5NF. Tel: (0902) 892141.

Submissions, which must reach me by 1 June 1986, will be judged using suitably vague criteria. A prize will be awarded for the best entry received.

*Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.*

*Expanded revues of previous problems together with, subject to the approval of the contributor, copies of detailed programs from the winning entry may also be requested. However, in the interests of efficiency, interested readers are urged to contact the prize-winner directly.*

## *Diophantine Equations — September 1985 review*

This problem gave rise to a number of very well presented submissions. Space does not permit the listing of the full solutions obtained to Case 1, order A, B & C; these are to be seen in the recent paper by Leo J Alex in *Mathematics of Computation,* vol 44, no 169, January 1985 pp 267-278, and may be obtained from me (at the above address) if required.

After considerable thought it was decided that this month's prize-winner should be Ghislain Deridder of 23, The Glades, Worsted Farm Estate, East Grinstead, RH19 3XW. Ghislain used his recently acquired Amstrad CPC 6128 in Amstrad Locomotive Basic and explored numerous Case 2 problems in addition to complete analysis of Case 1. The programming was not very ambitious but nonetheless functional, and run times were quite realistic; the submission included a discussion of the limitations of the work submitted and possible ways of overcoming these.

Perhaps the prize will help Ghislain in some way towards the acquisition of a printer for the system, although the listings he submitted were carefully copied and should encourage other readers without a printer to try their luck with a submission!