NUMBERS COUNT

END ZONE

Following up on the enthusiastic response given to Mike Mudge's musings on Perfect and Tri-perfect Numbers, this month he captivates you further with Sociable Numbers.

Definition (i). The *Aliquot Divisors* of a positive integer, n, are all of its divisors except for n itself. s(n) is used to denote the sum of these aliquot divisors of n.

(ii) If s(n) = n, then n is said to be a *Perfect Number* (see 'Numbers Count', October 1983 & March 1984).

For example: if n = 28, then the aliquot divisors are 1, 2, 4, 7 and 14 from which it is readily seen that s(28) = 28.

(iii) If s(m) = n and s(n) = m, then m and n are said to form an *Amicable Pair.* In this case $s(s(n)) = s^{(2)}(n) = n$, where s is considered as an operator acting upon a positive integer.

For example: if n = 220, then the aliquot divisors are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, so that s(220) = 284. The aliquot divisors of 284 are 1, 2, 4, 71 and 142 whose sum is 220, thus $s^{(2)}(220) = 220$, and 220 and 284 are an amicable pair.

(iv) If s(m) = 2n, then n is said to be *Triperfect* (see 'Numbers Count' November 1984, May & July 1985).

The operator s can now be iterated; that is, applied repeatedly to a given positive integer, n.

(v) The numbers in the set $n_1, n_2, n_3, \ldots n_k$ are called *Sociable Numbers* with *Index* k, if and only if:

$s(n_1) = n_2$
$s(n_2) = n_3$
....
$s(n_{k-1}) = n_k$
$s(n_k) = n_1$

For example: 12496 1s is a Sociable Number with Index 4 because: s(12496) = 14288, s(14288) = 15472, s(15472) = 14536 and s(14536) = 12496.

It can be readily verified that 14316 is a Sociable Number with Index 28.

Until 1918, two sets of sociable numbers with indices 5 and 28 were known; reference P Poulet, 1918, *Intermed Math* (vol 25, page 121). H Cohen, reference 1970, *Math Comp* (vol 24, page 423) has conducted an extensive search for sociable numbers with index less than, or equal to, 10, of which the smallest member of the set is less than 60000000. This search has revealed nine new sets of sociable numbers with index '4' and has led Cohen to the interesting conjecture that there exists an infinity of sociable numbers with index 4.

Problems (A) Rediscover Cohen's nine sets;

(B) Iterate the function s(n) for any arbitrary starting integer n_0, and hence find the index of sociability of n_0; and

(C) Generalise the concept of sociability in some way: for example, by redefining the function 's' as the sum of the even (or indeed the odd) aliquot divisors; or perhaps excluding aliquot divisors of the form p^n where p is prime.

Readers are invited to submit their attempts at some (or all) of the above problems to: Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Wolverhampton, Staffordshire WV4 5NF. Tel: (0902) 892141.

Submissions, which must reach me by 1 August 1986, will be judged using suitably vague criteria. A prize will be awarded for the best entry received.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. Expanded revues of previous problems, together with, subject to the approval of the contributor, copies of detailed programs from the winning entry, may also be requested. However, in the interests of efficiency, interested readers are urged to contact the prizewinner directly.

November 1985 review

Samuel Yates, 1975, published Prime Period Lengths for the first 105000 primes up to:

Prime Number	Period Length
1370329	342582
1370359	228393
1370377	152264
1370389	456796
1370407	1370406
1370431	685215
1370449	685224
1370459	1370458
1370461	274092
1370471	685235

These tables, referred to by their author as 'a silo or granary of food for thought', were followed by numerous conjectures, some of which were included in the November 1985 issue.

The competition winner this month is Ronald B Shepherd, of 2 Orchard Croft, Cottingham, Humberside HU16 4HG, for a very careful piece of work carried out using The Modula-2 compiler from Hochstrasser Computing AG, Geroldswil, Switzerland, on a Sharp MZ-80B with twin 5.25in disks and a printer. Ronald has combined efficient programming with excellent presentation and has reported to certain statistical tests on the hypotheses provided in *PCW.* He will, I am certain, be very willing to discuss his findings directly with any interested readers.

LEISURE LINES

Brain-teasers courtesy of JJ Clessa.

[illegible] 98624173
[illegible] 98246713

Quickie

Which temperature reads the same in degrees Celsius as it does in Fahrenheit? (No postcards, please.)

Prize puzzle

May 86

Consider the sequence of prime numbers: 3, 31, 431, 5431, 54319, and so on.

Each successive prime number is formed by adding one digit to the front or to the end of the preceding number. Each digit added is different from those already present.

What is the largest possible prime number that can be obtained by this procedure?

Leading zeros are not permitted, so it should be clear that the largest prime possible cannot contain more than nine digits.

Answers (including the sequence used) on postcards only please, to