MICROCHESS

27	e4–e5	Rf8xa8
28	Qc3–c6	Ra8–b8
29	Kg1–f1	Bh3–f5
30	Qc6xc7	Rb8–c8
31	Qc7xd6	Qg5–d2

White has picked up two pawns for the piece, but this is killing.

31	Re1–b1	Bf5xd3+

0–1 (White resigns)

NUMBERS COUNT

Mike Mudge deals with PAPs (Primes in Arithmetic Progression), and presents the winning solution to Euler's Constant problem.

Definitions (i) A *prime number* is a positive integer which is only exactly divisible by itself and unity (one): for example, 2, 3, 5, 7, 11, 13, 17, 19, 23, . . .
(ii) A *palindromic number* is a number which reads the same backwards as forwards: for example, 1234321, 78000087.
(iii) An *arithmetic progression* is a sequence of numbers, each member differing from the previous one by the same constant quantity: for example, 7, 10, 13, 16, . . . or, in general, a, a+d, a + 2d, a + 3d, . . . Weask the question: how long can an arithmetic progression be, which consists only of prime numbers (a PAP)?

It is conjectured that a PAP can be as long as we wish. The truth of this conjecture would readily follow from an improvement to a theorem of Endre Szemeredi (see *Acta Math Acad Sci Hungar*, vol 20 1969, pp 89–104).

Sierpinski defines g(x) to be the maximum number of terms in a PAP not greater than x. The least x, 1(x), can then be regarded as a function of g(x) yielding the following table:

g(x) 2 3 4 5 6 . . .
1(x) 3 7 23 29 157 . . .

The first column refers to the PAP 2, 3, while the fifth column refers to the PAP 7, 37, 67, 97, 127, 157.

It has been conjectured that there are arbitrarily long PAPs of: (a) consecutive primes such as 251, 257, 263, 269, and 1741, 1747, 1753, 1759; and (b) palindromic primes such as 13931, 14741, 15551 and 16361.

Paul Erdös has broadened the problem by conjecturing that if (a_i) is any infinite sequence of integers for which the sum of the reciprocals is divergent, then the sequence contains arbitrarily long arithmetic progressions. He offers a prize of $3000 for a proof or disproof of this conjecture.

Problem A List all the PAPs of a given length contained within a given table of prime numbers.

Tabulate separately those consisting of: (a) consecutive primes; and (b) palindromic primes.

Problem B Extend Sierpinski's table of 1(x) as a function of g(x), printing the PAP to which each entry corresponds.

Problem C Investigate the Erdös conjecture for various sequences (a_i) whose reciprocals have a divergent sum.

Readers are invited to submit their attempts at some (or all) of the above problems to: Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Wolverhampton, Staffordshire WV4 5NF. Tel: (0902) 892141.

Submissions, which must reach me by 1 September 1986, will be judged using suitably vague criteria, and a prize will be awarded to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.

Expanded revues of previous problems, together with, subject to the approval of the contributor, copies of detailed programs from the winning entry, may also be requested. In the interests of efficiency, interested readers are urged to contact the prize-winner directly.

The writer (Mike Mudge) welcomes correspondence on any subject within the areas of number theory and other computationally related mathematics, and will endeavour to reply to all letters after sufficient time has elapsed!

December review

Euler's Constant, defined as the limit as n, tends to infinity of (1/1 + 1/2 + 1/3 + . . . + 1/n) − $\log_e n$ = 0.5772156649015328606065 1209 . . . (see for this, and many other interesting numbers, *A Handbook of Integer Sequences* by NJA Sloane, Academic Press 1973).

The conjecture that 4/n = 1/x + 1/y + 1/z could be solved in positive integers for all n greater than 1, has been verified for n less than or equal to 10^8 by Nicola Franceshine; the corresponding result for 5/n is explored up to n = 1057438801 by Stewart, who covers all n *not* of the form 278460k + 1.

There are many other problems — for example: 'What is known about sets of unequal, odd integers whose sum is unity, such as 3, 5, 7, 9, 15, 21, 27, 35, 63, 105, 135?' The interested reader is referred to *Diophantine Equations* by LJ Mordell, Academic Press 1969, and also to *Unsolved Problems in Number Theory* by RK Guy, Springer 1981.

The prize-winner this month is Henry Ibstedt of 4 Rue Gramme, 75015 Paris, who, in addition to suggesting that Folkman's Number is probably bigger than Skewes' Number, used his IBM PC with 256k RAM in Basic to discover 14 140-digit equal denominators in the sums S_{317} to S_{337}, and extended the search to S_{457}. No equal numerators exist between S_1 and S_{457}.

Henry implemented two methods of representing integers with different terms from the harmonic series, expressing 5 as the sum of 1920 such terms and 6 as the sum of 1658880 such terms.

Many other results and investigations were detailed in an extremely well-documented submission, resulting in a most worthy prize-winner.

Well done, Henry.

LEISURE LINES

Brain-teasers courtesy of JJ Clessa.

No answers — no prizes. Which number, when multiplied by three, gives the same result as if it were added to 20?

Prize puzzle

The four-digit number 4151 has the property that the number is equal to the sum of the fifth powers of its digits — 4151 = $4^5 + 1^5 + 5^5 + 1^5$.