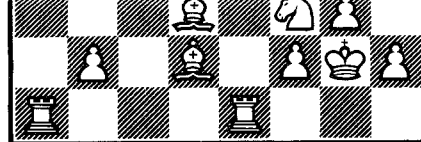
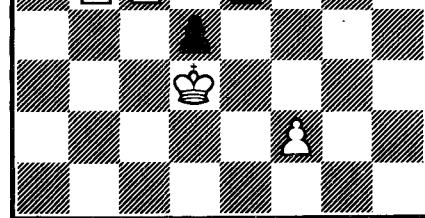


7 ... Bf5×b1
In a very simple position, Black plays a move that no human would



19 ... a7-a6
Black's task is extremely difficult, but this makes it easier for White to convert his pawn majority.
20 b4-b5 a6×b5
21 Bd3×b5 h7-h6
22 b2-b4 Kg8-f8
23 h2-h4 Kf8-e8

This is the end, losing material to the following, positional, combin-



42 ... 1-0
(Black resigns)

White not only has the overwhelming material superiority of bishop v pawn, but the connected passed b and c-pawns are bound to force their way through to queen.

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Mike Mudge examines S_k — sets and their extension.

Sets (ti) for which $t_i t_j + k$ is always a square. . . S_k — sets and some possible extensions

Definition (i) An S_k — set of size n is a set (t_1, t_2, \dots, t_n) of distinct positive integers such that $t_i t_j + k$ is the square of an integer whenever $i \neq j$; k being constant.

For example, $(1,2,5)$ is an S_{-1} — set of size 3 since $1 \times 2 - 1 = 1^2$, $1 \times 5 - 1 = 2^2$ & $2 \times 5 - 1 = 3^2$.
 $(1,79,98)$ is a P_2 — set of size 3 since $1 \times 79 + 2 = 9^2$, $1 \times 98 + 2 = 10^2$ & $79 \times 98 + 2 = 88^2$.

Definition (ii) AC_k — set of size n is

similarly defined using the condition that $t_i t_j + k$ is the cube of an integer whenever $i \neq j \neq p$.

It should be observed that these definitions are capable of modification in many natural ways; typical illustrations being
(a) $t_i t_j + k$ is the cube of an integer,

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(b) $t_i + t_j + k$ is the square of an integer,
(c) $t_i + t_j + t_p$ is the cube of an integer.

Definition (iii) An S_k — set is 'extendable' if there exists a positive integer, y say, not a member of S_k such that the union of y and S_k is still an S_k — set.

For example, the S_1 — set $(1,3,8)$ of size 3 can be extended using the integer $y = 120$ to generate the S_1 — set $(1,3,8,120)$ of size 4.

It has been shown (A Baker and H Davenport, *Quart Journal Math Oxford Ser (3)* v 20, 1969, pp129-137) that no further extension of this S_1 — set is possible.

The extendability of C_k — sets is similarly defined.

Problem A Catalogue, according to their size, all possible S_k — sets with elements less than some given N_0 .

Problem B Investigate the 'extendability' of these S_k — sets using integers y up to some given Y_{\max} .

Problem C Repeat (A) & (B) above for C_k — sets. . . of which none are known to the author.

Problem D Modify definitions (i) & (ii) above and attempt (A), (B) and (C) as appropriate.

Some reference to the theoretical

literature on these matters may be helpful. Details of the history of this problem are to be found in P Heichelheim's; *The Study of positive integers (a,b) such that $ab + 1$ is a square*. Fibonacci Quarterly. v17, 1979, pp269-274, also LE Dickson; *History of the Theory of Numbers*, vol II, pp513-520.

Readers are invited to submit their attempts at some (or all) of the above problems to: Mike Mudge, 'Square Acre', Stourbridge Road, Penn, near Wolverhampton, Staffordshire WV4 5NF, tel (0902) 892141. Submissions, which must reach me by 1 October 1986, will be judged using suitably vague criteria, and a prize will be awarded to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.

Expanded reviews of previous problems, together with, subject to the approval of the contributor, copies of detailed programs from the winning entry may also be requested. In the interests of efficiency,

interested readers are encouraged to contact the prize-winner directly.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computationally-related mathematics, and will endeavour to reply to all letters.

January review

This subject area produced responses ranging from 'What a load of rubbish!' to 'In order to produce a genuine program to perform two-way arithmetic, it seems one would have to start from Peano's axioms and communicate with the computer in machine-language. . .

The subject is self-explanatory; readers requiring further background and state-of-the-art reports are encouraged to take out a subscription to *Colsen News* (two-way numbers) with Cedric AB Smith Cedric Smith at 141 Portland Crescent, Stanmore, Middlesex HA7 1LR.

Detailed computer programs received were minimal. This month's prize-winner is A Sumner of 14 Western Elms Avenue, Reading RG3 2AN who has already received a complimentary copy of *Colsen News*.

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