

Mike Mudge goes back to basics . . . how large is a large integer? He also looks at the results, and some of your ideas, to April's Geometry of Numbers problem.

The primary requirement in empirical number theory is to escape from 'the largest integer which can be handled is $2^{32} - 1$ ' ($= 4294967299$) or similar restriction to be found in the user manuals of all personal computers.

Now, what is meant by 'handling' a large integer?

(a) Identification Ideally using a single identifier, as with conventional integers (maybe prefixed by L for 'long'), but possibly using an array-type declaration.

(b) Input and output It is essential to provide a facility enabling a large integer to be input to memory. Such a facility may require the integer to be accompanied by a parameter defining its length, either in digits or some other convenient unit; and should contain a sum-check to detect the majority of keying errors.

The output of a large integer need not be accompanied by the length-defining parameter: however, attention should be paid to formatting in relation to the output device in use. Spurious line-filling characters should be avoided and the conventional form of right justification is desirable, although not essential

(c) Very simple arithmetic Addition and subtraction of large integers are to be readily available, called through either user-named functions (or procedures) or ideally in the natural manner with + and - linking the appropriate identifiers.

(d) Simple arithmetic Multiplication of large integers should be carried out with any possibility of unreported overflow removed; in general, reported overflow (*Note: since all computers have a finite capacity, overflow is ultimately unavoidable*) must be accompanied by termination of the program.

(e) Not-so-simple arithmetic Division: we are unable to subscribe to the philosophy of the eminent number theorist DH Lehmer — 'Not only should division by zero be excluded from arithmetic, but all division should be avoided. . .'. Division should produce a quotient and a remainder, both of which may be very large: a trivial example 1234567898/231 yields 3846005 remainder 293. Decimal places are *not* relevant and floating-point arithmetic should be avoided in general.

Partial factorisation is a desirable

standard subroutine, assuming the accessibility of a 'table' of prime numbers up to a given maximum. An efficient division routine with a test for non-zero remainder will clearly expedite this procedure.

Determination of integer roots (or rational powers). The determination of square, cube, fourth. . . roots of large integers — *only when these are integers* — is a desirable facility and can clearly be related to the factorisation algorithm referred to above. For example, $7\sqrt{(105413504)} = 7\sqrt{(2^7 \cdot 7^7)} = 2 \cdot 7 = 14$.

A Computation of Devi The following 201 — digit integer had its 23rd root extracted mentally by Miss Shakuntala Devi in 50 seconds. A Univac 1108 computer at The National Bureau of Standards, Washington, using 13,000 pre-programmed instructions, repeated the calculation in one minute!

91674867692003915809866092758538
01624831066801443086224071265164
27934657040867096593279205767480
80679002278301635492485238033574
53169351119035965775473400756816
88305620821016129132845546805780
158806771

Specimen problem

Input the large integer 12345678910112131415. . . n, calculate its n^{th} power and output this, together with its factors. What is the maximum n which your computer will handle? For example, $n = 2$, $12^2 = 144 = 2^4 \cdot 3^2$, $n = 3$, $123^3 = 1860867 = 3^3 \cdot 41^3$.

Readers are encouraged to investigate the efficient manipulation of large integers using techniques ranging from array facilities in high-level languages such as Basic, Pascal or Fortran, through general string handling to machine code/assembler coding. The specimen problem together with the computation of Devi should be attempted.

Achievements may be submitted to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, nr Wolverhampton, Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 January 1987. It would be appreciated if such submissions contained a brief summary of results, together with thoughts relating to the associated outstanding problems, in a form suitable for future publication in PCW.

These submissions will be judged using suitably vague criteria, not nec-

cessarily related to the answer to Devi's Computation! A prize will be awarded to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computationally related mathematics, particularly containing suggestions for future 'Numbers Count' articles, and will endeavour to reply to all letters after a sufficiently long time.

Interested readers can be readily put into contact with others who have submitted attempts at previous problems. However, in the interests of efficiency, readers are encouraged to contact the prize-winner directly.

Review — The Geometry of Numbers

Relatively few graphics enthusiasts answered the call of April's problem. The typographical error in question five, line three — the word 'lattice' should be deleted — brought rapid response from several careful readers; however, question five, problem E, proved to be too abstract. Further comments relating to it would still be very welcome.

Any reader interested in the mathematical background of this topic is encouraged to consult Minkowski's classic work, *The Geometry of Numbers*. Those particularly interested in question four, problem C, are referred either to C Hylten-Cavallius, 'On a combinatorial problem', *Colloq Math*, vol six, 1958, pp 59-65, or more readily to Robin Merson, 2 Vine Close, Wrecclesham, Farnham, Surrey GU10 4TE, tel: Frensham 3587, who has extended the results quoted to $k(8) = 43$, $k(9) = 50$, $k(10) = 61$ and $k(15) = 121$, with marginal assistance from his Apple computer.

Richard Tindall of 26 Poplar Close, Great Shalford, Cambridge, a regular correspondent, has dealt effectively with problems A and B, and having confirmed the known results of problem D, has conjectured (without proof) results for $n = 8$ and 9. Problem C was Richard's chief interest and I feel sure that he would share

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the details of his approach upon receipt of a stamped, addressed envelope.

The prize-winner this month is John Gale of 16 Wrensfield, Hemel Hempstead, Herts HP1 1RN, tel:

(0442) 53643, who used a Video Genie with an Epson MX80 and programmed in Microsoft Basic. John's approach to graphical display of lattice points was simple but effective, and his attempts to re-

late results in the geometry of numbers to rational approximations (best-possible in some sense) to certain transcendental numbers were enthusiastic, if not greatly productive.

LEISURE LINES

Brain-teasers courtesy of JJ Clessa.

Of 100 people interviewed, 10 said they neither smoked nor drank, 75 said they drank, and 83 said they smoked.

How many people both drink and smoke?

Prize puzzle

When someone shouted 'Bingo', Fred still had nine numbers to get on his card. There was one number in each column, and three on each of the three rows on the card.

Fred noticed that one row contained three square numbers, one

contained two squares and a prime, and the other contained one square and two primes. Moreover, the total of the three numbers in each row was the same.

What were the numbers, and which rows were they on?

(Note: the Bingo card contains 9 × 3 squares, and the first column contains numbers less than 11. The second column has numbers in the range 11-20, the third 21-30, and so on.)

Answers on postcards, please, or

backs of envelopes, to reach us not later than 30 October 1986. Send your entries to Leisure Lines, October Prize Puzzle, PCW, 32-34 Broadwick Street, London W1A 2HG.

July prize puzzle

Not too difficult a problem.

The answer to the puzzle is that there are 191 numbers (excluding unity) which have no common factors with the number 720. The winning entry came from Paisley, and was from Mr John Paton. Congratulations John, your prize is on its way.

ACC NEWS

The ACC will be at the PCW Show — find out what it can offer you, courtesy of Rupert Steele.

The Association of Computer Clubs acts as the national umbrella organisation for computer clubs around the UK. This month, the ACC has been actively involved in preparations for the PCW Show, setting up the club area there. This has been coordinated as usual by Dr David Annal, who has run the hobby club presence at the show for several years; indeed, few can remember a PCW Show without the ACC's presence. We remain grateful to PCW and the show organiser, Montbuild, for their continued sponsorship of this aspect of the ACC's work, and believe that the show will again be enhanced by the reserve of unbiased help and advice that the hobby stands provide.

Outside PCW show time, the ACC provides a number of other services to clubs. An important aspect of this is the insurance scheme, whereby each qualifying computer club is covered against public liability risks as part of its affiliation fee, and through which inexpensive insurance is available for equipment at club meetings. The ACC also runs the ClubSpot 810 database on Prestel, which is constantly one of the most accessed parts of the whole system, and is expanding into new areas on Prestel,

including hobbies more generally. This is all run through the electronic publishing sub-committee of the Association, which reports to the Council of computer club representatives from around the UK.

The ACC also helps computer clubs with their publicity, both through this column and through our national clubs database, through which people can get in touch with their local computer club. To help keep the costs of affiliation down, the ACC sells label sets of the database to those considering direct mail to computer clubs.

Around the clubs

Alan Sharkey has written to me about The Anglia PC User Group. This is a new club, presumably for PC (that is, IBM think-alike) users in East Anglia. The group recently had its first meeting, which was apparently a considerable success. United Business Systems of Norwich provided the premises for the first two events. The first meeting featured Compaq, complete with some highly entertaining John Cleese videos. This was followed up by X-Data Ltd. As well as the meetings, members are offered public domain software, regular newsletters, cheap diskettes, and help with

problems. For details, contact Alan on (0223) 244438 (evenings) or (0493) 730107 (daytimes), or you can write to him at 5 Forest Road, Cherry Hinton, Cambridge CB1 4JA.

Philip Ramage, the secretary of the Croydon Apple User Group, has very kindly been keeping me informed of what is going on in his club by sending me the monthly newsletter. A particularly interesting recent meeting featured the Pagemaker software for the Apple Mac. This has, of course, been hyped-up by the press everywhere — the basic idea is that with the Mac's onscreen icon environment, and using a relatively cheap laser printer, you can produce high-quality magazine-style pages 'in-house' without expensive typesetting equipment. However, Philip regrets that the kit is not quite cheap enough yet: 'One day, perhaps, this newsletter will be a multi-font publication with hi-res pictures of indiscretions from the last meeting, but for the time being you'll have to make do with a worn-out ribbon and the Gestetner, and just imagine the rest.' The club also intends to get an early demonstration of Apple's new machine. The club meets at the Waddon Hotel, 2 Stafford Road, Croydon, every third Thursday at 7.30pm. Or