# NUMBERS COUNT

*Mike Mudge invites you to pit your wits against the 'postage stamp problem', and presents the winning Sociable Numbers from the May issue.*

The particular postage stamp problem we are concerned with is as follows: an envelope may carry no more than s-stamps, there are available p-integer valued stamp denominations.

Given s and p, find the maximum integer n; (which depends upon s and p, thus we write n=n(s,p)), such that all integer postage values from 1 to n can be displayed upon the envelope. Further, determine all possible sets of p-stamp denominations satisfying this condition.

An equivalent problem, which is often encountered in the literature, includes a stamp of face value zero and then requires that each envelope carry exactly s-stamps. Notice that here, any shortfall from s with the p-integer valued stamps is made up with zero valued stamps, which are not real and hence do not appear in the count of p.

This may appear confusing, but hopefully the following examples clarify the situation:

For example:

**(i)** s=2, p=3. Here n(2,3)=8. The unique solution set is given by (0, 1, 3, 4) and envelopes are stamped thus: 0+1=1, 1+1=2, 0+3=3, 0+4=4, 1+4=5, 3+3=6, 3+4=7 and 4+4=8. We have displayed all integer postage values from 1 to 8 using exactly two stamps (including the zero value stamp where needed) drawn from the solution set.

**(ii)** s=2, p=6. Here, n(2,6)=20. There are now five solution sets: (0,1,2,5,8,9,10); (0,1,3,4,8,9,11); (0,1,3,4,9,11,16); (0,1,3,5,6,13,14); and (0,1,3,5,7,9,10).

**(iii)** n(1,m)=m with unique solution set (0,1,...m).

**(iv)** n(t,1)=t with unique solution set (0,1).

It has been shown that n(h,2)=entier $((h^2+6h+1)/4)$, where entier defines the smallest integer not greater than, for example, entier (3.14)=3.

If h is odd, then the unique solution set is (0,1,(h+3)/2). However, if h is even, there are two solution sets (0,1,(h+2)/2) and (0,1,(h+4)/2).

The only other case for which a nearly complete solution is known to the author is that for which p, the number of stamp denominations available, is 3.

G Hofmeister has shown that: $(4/81)s^3 + (2/3)s^2 + (66/27)s \leq n(s,3) \leq (4/81)s^3 + (2/3)s^2 + (71/27)s - 1/81$ if s is greater than or equal to 34.

It has been further suggested, by RK Guy, that for s large enough, n(s,p) is given by a finite set of polynomials in s of degree p.

**Note** A polynomial of degree n in x is simply an expression of the form: $a_0 + a_1x + a_2x^2 + a_3x^3 + \dots a_nx^n$: where $a_0$, $a_1$, ....$a_n$ are known coefficients.

Thus the solution for n(h,2) given above may be written: $n(h,2) = (h^2 + (3+3c)h + d)/4$ where $c=d\equiv h$, mod 2.

Recall that $p\equiv q$ mod r means that p and q leave the same remainder when divided by r: we read 'p is congruent to q modulo r.'

Guy's conjecture for p=3 is that for s greater than 19: $n(s,3) = (4s^3 + 54s^2 + (204 + 3c_r)s + d_r)/81$ where $c_r$ and $d_r$ are given for $s\equiv r$ mod9 (see Table 1).

The known results, which may be used as test cases for computer programs, are given in Table 2.

Interested readers are invited to investigate this postage stamp problem using any combination of 'trial and error', 'logical search' or algebraic analysis combined with suitable practical implementation.

The objectives include:

**(i)** the tabulation of results for n(h,2) ... an inherently trivial exercise using the given formula.

**(ii)** the numerical investigation of the Hofmeister inequality to display its actual content — that is, what does it really tell us about n(s,3) for a given s?

**(iii)** the conversion of Guy's table into actual values for n(s,3). How do these compare with the results of (ii) above?

**(iv)** the determination of n(s,p) for given s and p values ... using the known results quoted above as test cases and extending the empirical knowledge of this function as far as possible.

Readers are invited to submit their attempts at the above to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, near Wolverhampton, Staffordshire WV4 5NF, tel: (0902) 892141.

It would be appreciated if such submissions contained a brief summary of results, together with thoughts relating to this postage stamp problem in a form suitable for future publication in *PCW*.

Submissions, which must reach me by 1 February 1987, will be judged using suitably vague criteria, and a prize will be awarded to the 'best' contribution received.

*Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.*

## Sociable numbers

Several readers corrected the example of 12496 which is, in fact, a Sociable Number with index 5, since s(14536)=14264 and s(14264)=12496.

This was due to my copying, without verification, a result of Lokenath Debnath, *Number Theory with Electronic Computers, Int J Math Educ Sci Technol* 1982, v13, no5, pp603–617.

Colin Singleton of Sheffield should be congratulated on his eight-day search using a BBC Micro!

The sets of Sociable Numbers beginning with 1547860, 3317740, 3265940, 5753864 and 7538660 were revealed in a variety of locations including an IBM PC in Sweden and an Amstrad PCW8256 in London NW11.

A number of interesting unsolved questions have been posed as a result of investigations of this problem. Details are available on request.

A recent paper by W Borho and H Hoffman, *Math Comp*, v46, no171, pp281–293, generates 3501 new amicable pairs and is very readable, as the title 'Breeding Amicable Numbers in Abundance' suggests.

This month's prize-winner is Gareth Suggett of 11 Harrow Road, Worthing, Sussex BN11 4RB, who in addition to extensive numerical investigation and attempted generalisation, has studied the relevant results to be found in RK Guy's *Unsolved Problems in Number Theory*.

| r | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $c_r$ | 0 | 1 | 3 | 0 | −2 | 0 | 3 | 1 | 0 |
| $d_r$ | 46 | −81 | −1 | −170 | 0 | 62 | −26 | 0 | −154 |

*Table 1*

| s | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p | 3 | 4 | 10 | 13 | 3 | 5 | 7 | 10 | 3 | 6 | 3 | 6 | 4 |
| n(s,p) | 8 | 12 | 46 | 72 | 15 | 36 | 70 | 154 | 26 | 108 | 220 | 211 | 1094 |

*Table 2*