NUMBERS COUNT

Mike Mudge recognises the importance of reader participation in his column, so this month he sits back and listens while you do the talking.

Problem (i) (suggested by Professor Leo Alex of State Universtiy of New York) Solve in integers the equation
$1 + x + y = z$
where the primes dividing the product xyz are restricted to 2, 3 and 5. Professor Alex would like to assure readers that the number of solutions is finite!

Problem (ii) (suggested by the author, Mike Mudge) 1729 is the smallest number that can be expressed as the sum of two cubes in two different ways — that is:
$1^3 + 12^3 = 9^3 + 10^3 = 1729$

What is the smallest number that can be expressed as the sum of two cubes in three different ways? Notice that in 1912 W Lenhart obtained:

$$46969 = \left(\frac{95}{7}\right)^3 + \left(\frac{248}{7}\right)^3 = \left(\frac{149}{12}\right)^3 + \left(\frac{427}{12}\right)^3 = \left(\frac{341899}{30291}\right)^3 + \left(\frac{1081640}{30291}\right)^3$$

This is readily converted to an identity in integers by multiplying throughout by the common denominator $(7 \times 12 \times 30291)^3 = 2544444^3$.

How readily can the smallest number expressible in a — ways as the sum of b-terms, each the c^{th} power of an integer, be computed for realistically small a, b and c? Estimates of computing power needed?

Problem (iii) (suggested by Donald Cross of Exeter)
(a) 1729 is the smallest number that can be expressed as $a^2 + ab + b^2$ in four different ways with a & b positive (16-ways if negative a, b & c are allowed).

How many numbers less than a million can be expressed as the sum of two cubes in two different ways *and* as $a^2 + ab + b^2$ in *eight* different ways with a & b positive (generalise by removing the restraint less than a million.)

(b) If a numer is prime, can it be expressed as $a^2 + ab + b^2$ in more than one way with a & b positive?

Readers are encouraged to send their work, together with complete or partial attempts at the solutions to these problems, to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF, tel: (0902) 892141.

It would be appreciated if such submissions contained a brief summary of results; together with thoughts relating to these problems, in a form suitable for publication in *PCW*. These submissions will be judged using suitably vague criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions must arrive by 1 May and can only be returned if a suitable stamped, addressed envelope is provided.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or particular, for future 'Numbers Count' articles; all letters will be answered in due course.

Isolated readers can be put in contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner directly.

August review: Farey series

The subject of Farey Series produced an anonymous submission of Farey, and Farey 2. Would the programmer who recognises ScanPointer and PosCount and who quoted Theorem 29 from Hardy & Wright please contact me.

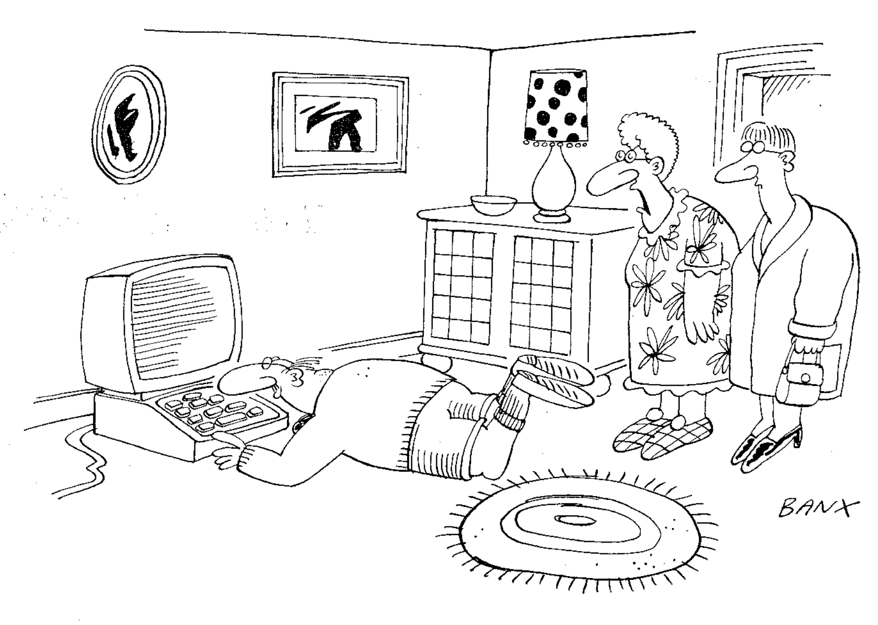
An estimate of 103 years, on a BBC Micro to construct F^{1025} using a crude bubble sort approach, is interesting . . .

A straightforward summary of Farey Series results together with a number of references is to be found in *Recreations in the Theory of Numbers* by Albert H Beiler (Dover 1964 £5.35) — a book which all 'Numbers Count' readers should possess.

Most attempts at this problem established simple arithmetical methods of generating terms of Farey Series and identified the major problem as one of satisfactorily displaying the output.

Graphical routines included those for a Tandy TRS-80, and a very efficient print routine for an Epson LX80 displaying F^{256} on, effectively, three sides of A4.

However after application of 'suitably vague criteria' (here it is important to emphasise that the purpose of the 'Numbers Count' competition is *not* to rank professional or semi-professional mathematicians or computer scientists, but to encourage empirical number theory as an alternative to game-playing on a personal computer; and to reward enthusiasm) this month's prizewinner is Ben Coffison of 9 McMurtrie Street, North Rockhampton, Queensland 4701, Australia. Ben programmed in Pascal, but due to local circumstances ran his programs on a VAX11 minicomputer; minor changes would make his routines available in TurboPascal. Ben would certainly welcome correspondence on this or related matters from *PCW* readers. END



'He's taken to his computer like a twelve-year-old.'