NUMBERS COUNT

Mike Mudge looks at W-Sequences, an introduction to an endless source of unsolved problems in number theory.

This problem area was first suggested to me by Philip Newton Webb of Llanelli, some eight years ago. At that time Philip had already spent seven years investigating a subset of the problems, and must be among the most experienced researchers into the properties of W-Sequences.

This topic provides a fascinating area for empirical number theory, a limitless supply of unsolved problems defined by an absolute minimum of mathematical symbolism, and I strongly recommend it as a natural entry point for new readers in this field.

The definition of a W-Sequence

Consider five positive integers a,b,c,d_1, and d_2 satisfying $2 \leqslant a \leqslant b$, $c \geqslant 0$, $d_1,d_2 \neq 0$.

The sequence $W(a,b,c,d_1,d_2)$ is defined by the following rules:

(i) The first term $W_1 = c$.

(ii) The even terms $W_{2n} = aW_n + d_1$.

(iii) The odd terms (other than the first defined at (i) above)

$W_{2n+1} = bW_n + d_2$.

(iv) The sequence calculated as above is then rearranged so that the terms are in increasing numerical order; thus, in general, the subscript n will no longer be in numerical order. Note: If $d_1=d_2=1$ we write $W(a,b,c)$, and if further $c=1$ we abbreviate the notation to $W(a,b)$.

Further, it should be observed that if $d_1=d_2$ then the value of a equal to b is excluded; we then have $2 \leqslant a < b$. Without this restriction, it is easy to see that $W_{2n}=W_{2n+1}$ and everything becomes rather trivial.

An example of a W-Sequence

If a=3, b=5, c=2 and $d_1=d_2=1$, then W(3,5,2) is generated as follows:

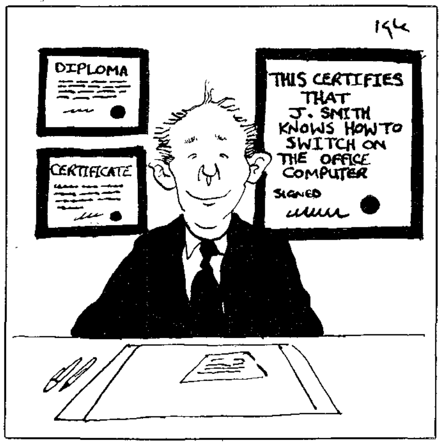
$W_1=c=2$; $W_2=3W_1+1=7$;
$W_3=5W_1+1=11$; $W_4=3W_2+1=22$;
$W_5=5W_2+1=36$; $W_6=3W_3+1=34$;
$W_7=5W_3+1=56$; $W_8=3W_4+1=67$

then rearranging we obtain: 2, 7, 11, 22, 34, 36, 56, 67, 103, 109, 111, 169, 171, 181 as far as W_{14}.

Junction points of a W-Sequence

For certain W-Sequences — that is, for certain choices of a,b,c,d_1 and d_2 — there exist *junction points* denoted by Z where $Z=W_m=W_n$ and the two subscripts m and n are not equal. For example, in W(2,6) we find: $W_1=1$, $W_2=3$, $W_3=7$, $W_4=7$; thus $Z_1=7$ is the first junction point.

In W(2,5) we find that
$W_7=W_{16}=31=Z_1$.
In W(2,3) we find that
$W_{11}=W_{16}=31=Z_1$
$W_{51} = W_{80}=175=Z_2$
. . .
$W_{35291}=W_{202832}=1640335=Z_{101}$



Problems

(a) What are the terms of a W-Sequence? Test cases: evaluate W(2,3), W(2,3,2), W(6,9), W(3,4,1) and W(3,4,2) between 1 and 10^6. Count the number of terms in each and further show the sub-totals for each 10000.

(b) Given n, evaluate W_n for a specified W-Sequence.

(c) What are the junction points, if any, for a specified W-Sequence? Test cases: evaluate junction points in W(2,3), W(2,3,2), W(2,3,c) where c is to be input.

(d) What are the values of a,b,c,d_1 and d_2 for which there exists at least one junction point?

Hints

(i) Apart from the value of $c=W_1$ itself, all terms in W(2,3,c) can only leave remainders 1,3,4,5 when divided by 6.

(ii) Apart from the value of $c=W_1$ and possibly W_2,W_3 and W_4, every term in W(6,9,c) leaves remainder 7,10,37 or 43 when divided by 54, except in the case where c is a multiple of 18 when the remainder 1 also occurs on division by 54.

Readers are encouraged to send their thoughts, together with complete or partial attempts at the solutions to the above problems, to Mike Mudge, Square Acre, Stourbridge Road, Penn, Staffordshire WV4 5NF, tel: (0902) 892141 to arrive by 1 July 1987.

It would be appreciated if such submissions contained a brief summary of results; together with thoughts relating to W-Sequences in a form suitable for future publication in *PCW*. These submissions will be judged using suitably vague criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a stamped, addressed envelope is provided.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or particular, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put into contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prize-winner directly.

Review: Back to basics

This invitation to go 'Back to basics' produced an excellent response; the transposition of 64 to yield 46 in the sixth line of Devi's Number being unfortunate, but not troublesome.

Submissions divided broadly into two classes: those who used string-handling software — for example, on a BBC Micro — and were restricted to 255-digit integers; and those who used the generally much slower array-handling software.

The winner has been chosen from the second category and is Ettrick Thomson of Woodhaven, Leiston Road, Aldeburgh, Suffolk IP15 5PX. Ettrick used a Spectrum Plus with 48k RAM, an Alphacom 32 printer and a cassette recorder; the normal Spectrum Basic being enhanced by BetaBasic written by Betasoft and allowing for example procedures with arrays as parameters.

It should be mentioned that Ettrick's programs were by no means the most efficient submitted, but some feel for his approach may be obtained from the following extract: '. . . printouts use old-style numerals which, like lower-case letters, have ascenders and descenders. As with lower-case letters, these help in avoiding confusion between certain numerals and certain numeral/letter combinations . . . It would be a pity if the USA influence led to a disappearance of old-style numerals, especially in work on large integers.'

Mention must be made of the submission from Alan Thomas of Tasmania who refers readers to his paper of January 1980 in *APC*, vol 1, no 8, page 64, detailing Monster Multiplier based upon The Trachtenberg Speed System of Basic Mathematics. Alan also has a Devi-ous Method for the Devi Calculation which I will be pleased to forward to readers. END