NUMBERS COUNT

Mike Mudge continues his investigation into W-Sequences.

In the April 'Numbers Count' the definition and origin of W-Sequences appeared together with a number of simple problems. For completeness the definition is reproduced here and a further, independent set of problems is then formulated.

The Definition of a W-Sequence

Consider five positive integers a,b,c,d_1 and d_2 satisfying $2 \leqslant a \leqslant b$, $c \geqslant 0$, $d_1,d_2 \neq 0$.

The sequence $W(a,b,c,d_1,d_2)$ is defined by the following rules:

(i) the first term $W_1 = c$.

(ii) The even terms $W_{2n} = aW_n + d_1$.

(iii) The odd terms (other than the first defined at (i) above)

$W_{2n+1} = bW_n + d_2$.

(iv) The sequence calculated as above is *then* rearranged so that the terms are in increasing numerical order. Thus in general the subscripts n will no longer be in numerical order.

Note: If $d_1 = d_2 = 1$ we write W(a,b,c) and if further c = 1 we write W(a,b). For an example of a W—Sequence see last month's column for detailed calculation.

W(2,3,1): 1, 3, 4, 7, 9, 10, 13, 15, 19, 21, 22, 27, 28, 31, 39, 40 . . .

W(2,3,2): 2, 5, 7, 11, 15, 16, 22, 23, 31, 33, 34, 45, 46, 47, 49, 63 . . .

W(2,3,7): 7, 15, 22, 31, 45, 46, 63, 67, 91, 93, 94, 127, 135, 136 . . .

Revision Note

Two positive integers A and B are said to be CONGRUENT MODULO a third positive integer C if and only if they leave the same remainder when divided by C, thus A — B must be an integer multiple of C.

We write $A \equiv B \pmod{C}$ and understand that there exists a positive integer k such that A — B = kC where it is assumed that $A \geqslant B$.

Problem I

What values may W(a,b) take modulo any given integer?

Hint. Produce a chart expressing the values of W(a,b) (mod ab) as a percentage for values $2 \leqslant a < b \leqslant 9$. Where the values to be charted are the RESIDUES MODULO ab — that is: the remainders upon division by ab.

Extend the chart as far as practicable. For example: $2 \leqslant a < b \leqslant 50$.

Extend the results to include W(a,b,c) (Mod ab) for $1 \leqslant c \leqslant ab$.

Problem II

What proportion of the terms of a W-Sequence are congruent to each of the possible residues modulo N?

Hint. Determine the proportion of the terms of W(2,3,1) which are congruent to each of 0,1,2 and 3 modulo 4 at intervals of, say, 20000.

Determine the proportion of the terms of W(2,3,1) congruent to each possible residue modulo 100 at intervals of, say, 20000.

Repeat each of the above for W(2,3,2) and then turn the investigation to W(6,9) modulo 54 at intervals of, say, 100000.

Why is it only necessary to consider 7, 10, 37 and 43 as possible residues?

Problem III

For what distinct values of c is a given N a term in $W(a,b,c,d_1,d_2)$ when a,b, d_1 and d_2 are specified?

Problem IV

What terms are common to $W(a_1,b_1,c_1)$ and $W(a_2,b_2,c_2)$ and are these terms all in $W(a_3,b_3,c_3)$ for some suitable choice of the parameters?

Hint. Examine the case of W(2,3,1); W(2,3,2) and W(2,3,7) for which a few terms are given above.

'It says: "Would you be interested in a little romantic novel I've written?" '

Readers are encouraged to send their thoughts, together with complete or partial attempts at the solutions to the above problems, to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF, tel (0902) 892141, by 1 August 1987.

It would be appreciated if such submissions contained a brief summary of results together with thoughts relating to these problems, in a form suitable for future publication in *PCW.*

Please note that submissions can only be returned if a stamped addressed envelope is provided.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or particular, for future 'Numbers Count' articles.

Review: November '86

The postage stamp problem attracted a wide spectrum of readers. Mention must be made of the experts based in Norway, whose correspondent was Christoph Kirfel of The Mathematics Institute, Universitetet I Bergen, Avd. B, 5014 Bergen, Norway.

Readers with detailed enquiries regarding the state of the art and mathematical background are encouraged to contact Christoph directly.

Readable, but slightly out of date references include: *A Postage Stamp Problem* by Ronald Alter and Jeffrey Barnett (American Mathematical Monthly, March 1980, pp206-210) with a further 47 references; *Algorithms for Computing the h-Range of the Postage Stamp Problem* by Svein Mossige (from Bergen) (Mathematics of Computation, vol 36, no 154, April 1981 pp575-582); and *Unsolved Problems in Number Theory,* by Springer Verlag 1980, p68-70.

Within the spirit of The Numbers Count column and its associated vague criteria, this month's prizewinner is Peter Cameron of 70 Godstow Road, Wolvercote, Oxford OX2 8NY who programmed a ZX Spectrum using Hisoft's Pascal Compiler and Devpac Assembler/Monitor. Included in Peter's results is a table of n(s,3) for $s \leqslant 50$ compared with the upper and lower Hofmeister Bounds and Guy's conjectured value.

Readers should note that Guy's Conjecture is now, in fact, a proven result.

For further information please contact Christoph, Peter or Mike Mudge.

Calling all C Programmers . . .

Michael Scott, of The National Institute for Higher Education, Dublin 9, has produced the MIRACL library (Multiprecision Integer and Rational Arithmetic C Library) which runs under the IBM PC (MS-DOS/PC-DOS). It comes with some 50-plus A4 sides of documentation and Michael is anxious 'for the widest possible distribution of this software to help me debug/improve it'. Any interested readers prepared to undertake field trials of this software and to report back on their experiences should either write to Michael at The School of Computing and Quantitative Methods of The NIHE or to the author, Mike Mudge