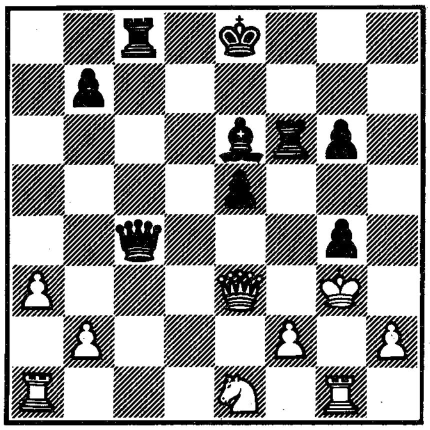
# MICROCHESS

been better.

| | | |
|---|---|---|
| 28 | ... | Bc4–e6+ |
| 29 | g2–g4 | Qc7–c4 |
| 30 | Rh1–g1 | h7–h5 |
| 31 | Qa7–e3 | |

The queen comes back, but Black has had two free moves to increase his attack.

| | | |
|---|---|---|
| 31 | ... | h5×g4+ |
| 32 | Kh3–g3 | |
| 32 | ... | Qc4–f4+ |
| 33 | Qe3×f4 | e5×f4+ |
| 34 | Kg3–h4 | |



This hastens the end, but 34 Kg3–g2 also loses to 34 . . . Be6–d5+ 35 f2–f3 (or 35 Kg2–f1 f4–f3 36 Rg1–h1 Bd5–c4+ 37 Kf1–g1 Bc4–e2 and White is completely tied up) 35 . . . g4×f3+ 36 Kg2–f2 (on 36 Ne1×f3, Rc8–c2+ wins everything) 36 . . . Rf6–b6 (this is important because it opens up the seventh rank) 37 b2–b4 Rb6–e6 and Black wins: for example, 38 Rg1–g5 Re6–e2+ 39 Kf2–f1 Bd5–c4 40 Ne1×f3 Re2–a2+.

| | | |
|---|---|---|
| 34 | ... | Ke8–f7 |
| 35 | Kh4–g5 | Rc8–h8 |
| 36 | 0–1 | |

White cannot avoid being mated by Rh8–h5. END

# NUMBERS COUNT

*Mike Mudge sets two different problems this month and asks readers to let him know which they prefer.*

This month's 'Numbers Count' displays two totally different types of problem in empirical number theory. Readers are, as usual, invited to submit attempts at solution to either (or both) of the problems posed; but are also invited to indicate which subject area they prefer, hopefully with some logical reasoning.

**Problem I:**

**The Left Factorial Function**

Recall that factorial n, where n is a positive integer, is defined by: n! = 1.2.3.4.5.6. . . . n thus 6! = 720, 10! = 3628800.

Further, 0! = 1 by definition.

Following D Kurepa we write the left factorial function thus:
!n = 0! + 1! + 2! + 3! + . .+(n−1)!
thus: !6=0! + 1! + 2! + 3! + 4! + 5!
hence: !6=154 and !11 = 4037914

Is !n ever divisible by n (without remainder) if n is greater than 2? The conjecture is that the highest common factor of !n and n! is 2.

Further following SS Wagstaff we write:
$B_n$ = !(n+1) − 1
$B_n$ = 1! + 2! + 3! + . . . +n!

*'I just love Bank Holidays, don't you, Squeaky?'*

and observe that 3 is a factor of $B_n$ if n is greater than 1, that 9 is a factor of $B_n$ if n is greater than 4 and that 99 is a factor of $B_n$ if n is greater than 9. How does this generalise?

**Problem II:**

**On a Congruence of Mok-Kong Shen**

Recall that two integers a and b are said to be congruent modulo a third integer c if and only if a − b is divisible (without remainder) by c; we write a ≡ b (modulo c), for example 98 ≡ 46 (modulo 13) because 98 − 46 = 52 = 4.13.

A Rotkiewicz (1984) asked for all solutions of the congruence: $2^{n-2} \equiv$ 1 (modulo n). Five solutions are known in the interval 3, $10^6$). The smallest is 20737 and the largest is 540857. What are the others?

Mok-Kong Shen (1986) has shown that there are infinitely many positive integers k such that the congruence $2^{n-k} \equiv 1$ (modulo n) has infinitely many solutions for n; however, it remains an open question whether there are infinitely many solutions for n for *all* positive integers k.

While realising that a computer can never find an infinite number of solutions to any problem, how would the solutions to Shen's congruence be efficiently calculated within a given interval for n?

Readers are encouraged to send their thoughts, together with complete or partial attempts at the solutions to either of the above problems, to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 September 1987.

It would be appreciated if such submissions contained a brief summary of results obtained in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

## *Review: December '86*

Readers wishing to pursue the connection between The Fermat Quotient and Fermat's Last Theorem are referred to *13 Lectures on Fermat's Last Theorem* by Paulo Ribenboim (Springer-Verlag 1979) while those interested in the computations of Brillhart, Tonascia and Weinberger mentioned in *PCW* (December 1986 page 250/51) should consult *Computers in Number Theory* edited by AOL Atkin and BJ Birch (Academic Press 1971, pages 213/222).

This month's prizewinner is Ray Davies of 33 Windrush Crescent, Barrow-in-Furness, Cumbria LA14 3UJ. Ray used Basic on his BBC and concentrated entirely on the algorithm for solving $a^{p-1} \equiv 1$ (mod $p^2$) for p prime and different values of a. p is restricted to being less than $2^{15}$ to avoid integer overflow and the submission contained, in addition to listings and output a significant amount of theoretical background.

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Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or particular, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put into contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner directly. END