LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie
If you write down every positive 2-digit number (that is, from 10 to 99), which digit will you have written the most number of times?

Jul 87

Prize puzzle
(1) Take a 4-digit palindromic number — that is, one which reads the same from right to left as it does from left to right.
(2) Reverse the digits and add the result to give a new number.
(3) Repeat step 2 with the new number until the result becomes palindromic.

To illustrate, suppose we have the number 3883;

3883 + 3883 = 7766
7766 + 6677 = 14443
14443 + 34441 = 48884

which is palindromic after three cycles only.

Two numbers, however, do not yield palindromic results even after 1000 cycles. What are they?

Answers on postcards, please, or backs of envelopes only, to reach *PCW*, Leisure Lines July 1987, 32-34 Broadwick Street, London W1A 2HG, no later than 31 July 1987.

April prize puzzle
A moderate response — over 100 replies. As usual there was a trace of ambiguity in the problem — does the digit zero follow the digit 9 in the definition of 'consecutive'?

We decided not, since the problem already stated '... digits 0–9 are used ...', which really precludes the ambiguity. Anyway, most entrants who realised the possible ambiguity, sent in the correct solution as well — which was 123341234.

The winning solution came from Scotland — from Mr D Poyner of Charlestown. Congratulations.

NUMBERS COUNT

This month Mike Mudge looks at Cyprian's Last Theorem.

This theorem is due to the Reverend DC Stockford of Downside Abbey, Stratton on the Fosse, Bath; acknowledgement is also due to Mr M Kochanski of 7 Courtfield Gardens, London SW5 0PA who has carried out significant empirical and theoretical studies related to the theorem.

Clearly $3^2+4^2=5^2$, the smallest integer-sided Pythagorean triangle, is familiar to many readers; however, it is less well-known, although equally trivial, that $3^3+4^3+5^3=6^3$.

The geometrical model of a cube with side 6 units, dissected into three smaller cubes with sides 3, 4 and 5 units respectively, is an interesting application of computer graphics. Clearly more than three portions have to be dissected and then some reassembled. What is the smallest number of parts needed?

Cyprian's Last Theorem

$$\sum_{r=1}^{k}(x-1+r)^k=(x+k)^k$$

has no solutions in positive integers other than x=3 with k=2 or 3.

Note. The notation on the left-hand side is simply shorthand for $x^k+(x+1)^k + (x+2)^k \ldots (x+k-1)^k$ there are k terms.

As an appetiser readers are first invited to find 64 consecutive positive integers, the sum of whose cubes is a perfect cube. It is known that only one such set exists. What about the sum of the n^{th} powers of 64 consecutive integers being an n^{th} power?

What about the sum of the n^{th} powers of k consecutive integers being an n^{th} power?

What about the sum of the n^{th} powers of two integers being an n^{th} power? ... Fermat's Last Theorem.

Readers are invited to send their thoughts together with complete or partial attempts at the investigations of the above questions to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 October 1987.

It would be appreciated if such submissions contained a brief summary of results obtained in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a stamped addressed envelope is provided.

Review: January 1987

This problem involving Bernoulli's Numbers, Euler's Numbers and the connection with Regular Primes can be further studied by reference to *13 Lectures on Fermat's Last Theorem* by Paulo Ribenboim (Springer Verlag 1979). It proved to be a very popular problem among regular contributors but did not appeal to new readers. Why was this so?

The very worthy prizewinner was John B Cook of 34 Joan Crescent, East Burwood, Victoria 3151, Australia. John used a Tandy TRS-80 Model 4P to compute N(X) and D(X), to find irregular primes as defined in the article, and to calculate and factorise E(X), the latter up to X=28.

Test data, together with much other interesting material is to be found in *A Handbook of Integer Sequences* by NJA Sloane (Academic Press 1973).

It must be recorded, however, that Geoff Lockwood of 254 Crystal Palace Road, London SE22 9JH computed Bernoulli Numbers up to the 200^{th} halting then because the computation was taking two hours per number.

Geoff, however, unfortunately did not have time to consider the computation of the Euler Numbers but obtained some rather interesting results comparing true Bernoulli Numbers with the asymptotic formula in Ribenboim's book referred to above.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or particular, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put into contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner directly.