Mike Mudge explains the concept of difference tables.

Many readers will already be familiar with the concept of difference tables. These tables arise in any introduction to numerical methods or, more simply, in the process of interpolation — central to the use of tabulated function values (now, alas, frequently replaced, with a consequent lack of understanding, by the use of the pocket calculator!).

Suppose that $y = f(x)$ is tabulated at equal increments, h, in the independent variable x; these x-values being denoted by x_0, $x_1 = x_0 + h \ldots x_n = x_{n-1} + h = x_0 + nh$ and the corresponding y-values by $y_n = f(x_n)$.

The first forward differences, dy, of y are defined by $dy_n = y_{n+1} - y_n$.

The second forward differences, d^2y, of y are similarly defined by $d^2y_n = d(dy_n)$.

This apparently elaborate algebraic notation is readily clarified by the following example. Suppose $y = x^3 + 1$ with $x_0 = 2$ and $h = 3$: the difference table begins as shown in Fig 1.

Clearly, the second differences of n^2 are constant and equal to 2.

Question Do there exist non-consecutive integers x_0, x_1, x_2, . . . such that the second differences of their squares are constant? Specifically, can that constant be equal to 2?

Answer Yes! For example (6, 23, 32, 39) see Fig 3.

Duncan A Buell, of the Supercomputing Research Center, 4380 Forbes Boulevard, Lanham, Maryland 20706, USA, has recently (1987) completely characterised such sequences of length 4 but states that the existence of such sequences of length 5 (and above) is still an open question.

He poses an intermediate step, which he calls problem B, seeking a sequence of five integers n_0^2 n_1^2 n_2^2. N, n_4 where n_0, n_1, n_2 are not consecutive such that their second differences are constant, say, c, and specifically with c = 2.

Problems

(i) Construct a computer program to input function values and print out, correctly formatted, the associated difference table up to the n^{th} differences.

(ii) Search for sequences of four squares such as (6,23,32,39) and (39,70,91,108) whose squares have second constant differences.

(iii) Extend (ii) to sequences of five integers in the pattern of Buell above.

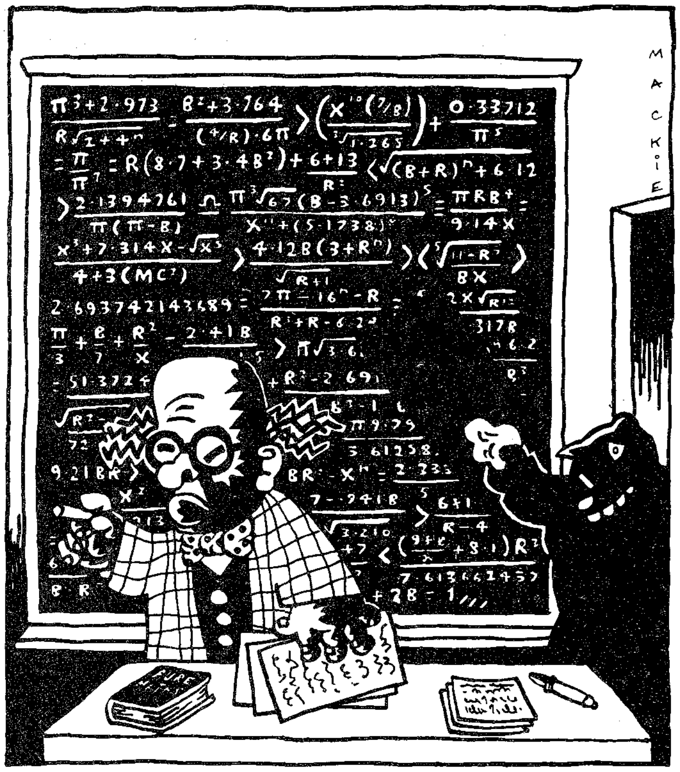
(iv) Attempt to resolve Buell's open question regarding sequences of five squares.

(v) Given that the n^{th} difference of a table of n^{th} powers is constant (see d^3y for $y = x^3 + 1$ above) investigate sequences of non-consecutive integers whose cubes have constant third differences, and so on, through fourth and fifth powers.

Readers are invited to send their attempts at some, or all, of the above problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel (0902) 892141, to arrive by 1 August 1988. It would be appreciated if such submissions contained a brief description of the program and a summary of the results obtained in a form suitable for publication in *PCW*.

These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.



x	$y = x^3 + 1$	dy	d^2y	d^3y
2	9			
		126 − 9 = 117		162
5	126		270	
		513 − 126 = 387		162
8	513		432	
		1332 − 513 = 819		162
11	1332		594	
		2745 − 1332 = 1413		162
14	2745		756	
		4914 − 2745 = 2169		
17	4914			

Fig 1

n	$y = n^2$	dy	d^2y
1	1		
		3	
2	4		2
		5	
3	9		2
		7	
4	16		2
		9	
5	25		2
		11	
6	36		

Fig 2 The difference table for n^2

i	n_i	$y = n_i^2$	dy	d^2y
0	6	36		
			493	
1	23	529		2
			495	
2	32	1024		2
			497	
3	39	1521		

Fig 3

Review, November

This problem produced a variety of responses, the largest powerful number seen being 467 9307774, degree 10, base 10. The geometrical interpretation hinted at in the article may well be a figment of the author's imagination — no-one made significant progress along these lines!

The very worthy prizewinner is Brian Stuart of Düsseldorferstr 11, 8000 Munchen 40, West Germany. Brian searches for powerful numbers for all number bases from 3 to 99 to all possible degrees, with a restartable algorithm. By 24 January 1988 he had reached 3×10^6 for all bases and 10^8 for some; with a target of 2^{31} 'at some 11 million per hour'.

Among the many interesting results were: (a) 19 5 16 base 24 (=11080 decimal) is powerful of degree 3 and the only powerful number base 24 less than 119×10^6; and (b) no powerful numbers found to base 90.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put in contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner.