Mike Mudge investigates prime residue indices and Artin's Constant.

Definition (1) Given a prime number p, the prime period length, L(p), is defined to be the number of digits in the period of the decimal expansion of the reciprocal of p (see 'Numbers Count', *PCW*, November 1985). **Note**: primes 2 and 5 are excluded from this discussion since their reciprocals generate finite and hence non-periodic decimal expensions.

Definition (2) Given a prime number p, the prime residue index, i(p), is defined by the quotient i(p) = (p − 1)/L(p).

For example, if p = 11, then 1/p = 0.090909 . . . *viz* 0.$\dot{0}\dot{9}$, and thus L(p) = 2, i(p) = (11−1)/2 = 5.

If p = 31, then 1/p = 0.$\dot{0}$32258064516129$\dot{}$ and thus L(p) = 15, i(p) = (31−1)/15 = 2.

Problem (A) For prime residue indices 1 to 100 (and beyond) determine the smallest prime, $p_{min.}$, possessing each index (see the example in Fig 1).

Residue indices have been discussed at length in, for example, *Studies in Mathematical Analysis and Related Topics*, Stanford University Press, 1962, pp202-210.

Definition (3) The fraction of all primes (excluding 3 & 5) having residue index i is denoted by A_i. Formally this definition may be written as shown in Fig 2.

Now Professor DH Lehmer has conjectured that for i greater than 1: $A_i = \frac{A_1}{i^2} \prod_{q|i} \frac{(q^2-1)}{(q^2-q-1)}$ where

(i) A_1, the fraction of all primes (excluding 3 & 5) having prime residue index 1 unity is known as Artin's Constant and has been determined empirically to be approximately 0.3739558;

(ii) π indicates the repeated product (thus $\prod_{q=5,6,7}(q+1) = (5+1)(6+1)(7+1)(=336)$; and

(iii) q|i indicates that the repeated product is to be taken over those factors corresponding to the *prime* divisors of i. For example:

$A_2 = \frac{A_1}{2^2} (2^2-1)/(2^2-2-1) = 3A_1/4$

$A_6 = \frac{A_1}{6^2} (2^2-1)/(2^2-2-1)((3^2-1)/(3^2-3-1)) = (A_1/36)(3)(8/5) = 2A_1/15$

Top to bottom: Figs 1-4

i(p)	1	2	3	4	5
$p_{min.}$	7	3	103	53	11

$A_i = \lim_{x \to \infty} \frac{\text{Number of primes less than or equal to x with residue index i}}{\text{Number of primes less than or equal to x}}$

(Limit as x tends to infinity)

i	A_i	B_i	$1 - B_i$	$1/(i+1)$
1	0.3739558			
2	0.2804669	0.6544227	0.3456	0.3333
4	0.0701167	0.7910204	0.2090	0.2000
6	0.0498608	0.8597758	0.1402	0.1429

i	1	2	5	6
Number predicted by Lehmer	3740	2805	189	499
Number counted	3755	2808	194	496

Problem (B) For i = 1 to 36 (and beyond) tabulate A_i, $B_i = \sum_{j=1}^{i} A_j$, $1 - \sum_{j=1}^{i} A_j$ and compare this final quantity with 1/(i + 1) (see Fig 3).

Problem (C) Using a convenient number of primes, count how many have a given residue index and compare the result with that predicted by Lehmer. Show statistically how the agreement improves with increasing numbers of primes. The target results based upon the first 10000 primes are shown in Fig 4.

Readers are invited to send their attempts at some, or all, of the above problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141 to arrive by 1 November 1988. It would be appreciated if such submissions contained a brief description of the program and a summary of the results obtained in a form suitable for publication in *PCW*.

These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received.

Please note that submissions can only be returned if a suitable SAE is provided.

Review, February: A Chess Board Problem

This somewhat novel area for a 'Numbers Count' problem generated considerable interest. Readers new to the problem are encouraged to read *Mathematical Puzzling* by Tony Gardiner, Section 26, pp121-124. The solutions for 'Queens' are:

n	5	6	7	8	9	10	11	12	13	14	15	16	17
f(n)	3	4	5	5	5	5	5	6	7	8	9	9	9

and, as an appetizer for further research, k(9) = 14 while k(11) = 21.

This month's prizewinner is Frank Webster of 125 Coniston Grove, Middlesbrough, Cleveland TS5 7DF, who used BBC Basic running on an Electron to investigate this problem.

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