

# NUMBERS COUNT

Some primes are rare, but some are very rare, as Mike Mudge explains.

E  
N  
D  
Z  
O  
N  
E

**Trivial thought:** the rarest type of prime number is the *even* prime number! There is only one and its value is 2.

**Sierpinski Primes** These are of the form  $p_s = n^n + 1$  and were first considered in 1958. Only two are known, 5 and 257. It is known that any others have more than  $3 \times 10^{10}$  digits. And there (infinitely) (m)any others?

**Wilson Primes** These satisfy  $(p_w - 1)! \equiv -1 \pmod{p_w}$ ; where  $n! = 1 \times 2 \times 3 \times 4 \times 5 \dots \times n$  and  $a \equiv b \pmod{c}$  means that  $a - b$  is an integer multiple of  $c$ .

Only three are known, 5 and 13 having been added to in 1953 (Goldberg) by 563. Keller has searched up to  $3 \times 10^6$ .

Are there (infinitely) (m)any others?

**Wieferich Primes** These satisfy  $2^{p_{wh} - 1} \equiv 1 \pmod{p_{wh}^2}$ . Only two are known, 1093 (Meissner, 1913) and 3511 (Beeger, 1922); in 1981 Lehmer searched up to  $6 \times 10^9$ .

Are there (infinitely) (m)any others?

**Cullen Primes** These are of the form  $p_c = n \times 2^n + 1$  and six are known: the corresponding values of  $n$  being 1; (Robinson 1958) 141; (Keller 1984) 4713, 5795, 6611 & 18496.

Are there (infinitely) (m)any more?

**Sophie Germain Primes** These are primes  $p_{SG}$  such that  $2xp_{SG} + 1$  is also prime.

**Note** If  $p$  is a Sophie Germain Prime then there are no integers  $x, y, & z$ , different from zero and not multiples of  $p$  such that:  $x^p + y^p = z^p$ .

Are there infinitely many?

**Associated Cullen Primes** These are of the form  $p_c = n \times 2^n - 1$  and six are known: the corresponding values of  $n$  being 2,3,6,30,75 & 81.

Are there (infinitely) (m)any more?

**Problem** Readers are invited to construct computer programs to investigate primes of the general form:

$$p = a \times b^n \pm c$$

Particular emphasis should be placed on  $c = 1, b = 2$  with values of 'a' including 3,5,7,9,11,17,19,21,33,45 & 57. However,  $a = k^2$  and  $a = k^4$  with  $b = 2$  and  $c = -1$ , also  $b=10$  and  $c = 1$  are of interest. **Note** with  $b = 2, n$  must be greater than 1, otherwise the well-trodden path of Mersenne Primes will be followed.

**Test data**  $17 \times 2^n + 1$  is prime when  $n = 3, 15, 27, 51, 147, 243 \dots$ ;  $45 \times 2^n - 1$  is prime when  $n = 1, 2, 3, 4, 5, 6, 8, 9, 14, 15 \dots$

**Problem II** What is the most efficient algorithm for listing the Sophie Germain Primes referred to above? How do these appear to be distributed within the set of all primes? Does this apparent distribution suggest

an infinite number of these (female) primes?

Readers are invited to send their attempts at these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 November 1988. It would be appreciated if such submissions contained a brief description of the program and a summary of the results obtained in a form suitable for publication in *PCW*.

These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

## Review, March

The results on the computation of  $p(x)$  are summarised by Paulo Ribenboim, *The Book of Prime Number Records*, Springer-Verlag, 1988, culminating with  $p(4 \times 10^{16}) = 1075292778753150$ .

This book provides the ultimate source of information, at the present time, for any reader interested in Prime Number results and related topics. Numerous readers computed  $p(x)$  up to at least  $x = 10^8 \dots$  with or without error, but there was little interest in the efficiency of the computation. Perhaps 'dummy costing' should be introduced into personal computing to provide a motivation for efficiency?

Despite Ribenboim's assertion: 'It should be noted that tables of primes on cards or tape are obsolete, because it is possible to generate all the primes  $p$  to any given bound ... quicker than one can read from card or tape', Russell Lavelle-Langham has offered all primes in order up to 50,000,000 on floppy disk at cost. Further details from 31 Risby House, Barleycorn Way, Limehouse, London E14 8DF.

The nearly pattern involving palindromic squares prompted a number of interesting replies, the most detailed of which was from David Poyner of 106 Waterhouse Moor, Harlow CM18 6BE. David defined an algorithm to derive successive values in the progression and established how the arrival of the impermissible *digit 11* at the eighth stage was responsible for the pattern breakdown. David is the worthy winner of this month's prize.

**Mike Mudge** welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put in contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner.

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