Prime suspect

Mike Mudge explains a persistence property of the positive integers resulting from the addition of their prime factors.

The problem to be investigated this month has been suggested by Paul Cleary, of Mexborough, South Yorkshire.

It is well known that any given positive integer can be uniquely represented as a product of prime factors. Having carried out this factorisation, the resulting factors are added to generate another positive integer and the process is then repeated.

For example, $117780 = 2\times2\times3\times5\times13\times151$; the sum of these factors is $176 = 2\times2\times2\times2\times11$; the sum of these factors is 19, which being a prime number will reduce no further.

Now, if the positive integers from 2 onwards are subjected to the above iterative process it becomes clear that very many reduce to 5, that is, the sum of the two smallest prime numbers 2 and 3.

For example, $148980 = 2\times2\times3\times5\times13\times191$ yielding $216 = 2\times2\times2\times3\times3\times3$, yielding $15 = 5\times3$ and in turn $8 = 2\times2\times2$ finally $6 = 3\times2$ hence 5.

Note: The prime numbers themselves reduce no further under this procedure so are omitted from consideration.

Paul Cleary has listed all the positive integers less than 10001 which reduce to 5 and has noted the occurrence in his list of a number of consecutive sequences (for example, 800, 801, 802, 803, 804) together with a 'sprinkling' of palindromes (such as 444, 484, 959).

Problem I Implement the above iterative procedure and examine in particular the sets of positive integers which reduce to prime numbers other than 5. In passing, it would be valuable to note the distribution of the persistence of the positive integers under this procedure, that is, the number of iterations needed to reach a prime number. This $p(117780) = 2$ while $p(148980) = 5$ from the above examples.

Problem Ia Repeat the above investigation but neglect the multiplicity of the prime factors, that is, add each distinct prime factor once only to obtain the positive integer for use at the next stage.

Problem II Consider an iterative procedure involving the sum of the squares (or indeed any other positive integer power) of the prime factors.

Note Careful consideration must be given to the question of the convergence (or termination) of the procedure for higher powers!

Problem IIa As II, but again neglecting multiplicities.

Readers are invited to send their attempts at some or all of these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 January 1989. It would be appreciated if such submissions contained a brief description of the programs and a summary of the results obtained in a form suitable for publication in *PCW*.

Review, May

Submissions relating to this problem squares of non-consecutive integers giving rise to difference tables having constant second differences, contained a considerable variety of material, ranging from almost random experimentation to very sophisticated algebraic analysis.

However, the prizewinner this month is Robin Merson, of 2 Vine Close, Wrecclesham, Farnham, Surrey GU10 4TE. Robin's submission extends to 15-plus pages of algebraic analysis together with extensive programming of his 'rapidly getting obsolescent' Apple II (indeed the associated Epson 80MX printer 'gave up' part-way through the investigation).

Related Reading

The attention of number theory enthusiasts is drawn to the recent publication of *Elementary Theory of Numbers* by W Sierpinski, editor A Schinzel, from North Holland Mathematical Library. This is the second, revised and enlarged English edition, of 1988. ISBN 0-444-86662-0, 513 pages in hardback only.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

Why will 1989 pennies be likely to fetch almost £20?

Prize Puzzle Nov 88

Not too difficult this month. When Harry began his new job he was told that his weekly wage, which was in excess of £40, would be increased by 99p every pay day. Harry had been paid a total of £407 since he started, and he was soon expecting to break the £60 a week barrier.

What was his starting wage?

Answers on postcards or backs of envelopes to arrive not later than 30 November 1988. Send your entries to: November Prize Puzzle, Leisure Lines, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG.

Prize Puzzle August

A reasonable response to this micro-solvable problem — almost 65 entries were received. The winning entry came from Mr DJ Allen of Penn, Bucks, who receives our congratulations. His prize will be on its way shortly.

The winning solution was: 770 combinations which exceed 100.

If you didn't win this time, don't give up — it could be your turn next.