PURSUITS

Mike Mudge's mathematical mysteries.

Much of the background to this month's problem is to be found in *Elementary Theory of Numbers* by W Sierpinski & A Schinzel, North Holland Mathematics Library, volume 31, 1988: essential reading for everyone interested in Number Theory.

Take three cubes

For the equation $x^3+y^3+z^3=n$, we seek integer solutions for x,y and z; the parameter n takes particular positive integer values.

Case 1 n=2. Here there are infinitely many solutions given by: $x=1+6m^3$, $y=1-6m^3$, $z=-6m^2$ where m is an arbitrary natural number; there are other solutions, however, which are not given by these formulae.

Case 2 n=3. Here there are solutions (x,y,z)=(1,1,1), (4,4,−5) (4,−5,4) and (−5,4,4). Are there any others?

Case 3 If n leaves remainder 4 or 5 when divided by 9 there are *no* solutions. (When n=4,5,13,14,22,23, and so on).

Case 4 n=6. Here there are solutions (x,y,z)=(−1,−1,2), (−43,−58,65) and (−55,−235, 236) together with their permutations by symmetry. Are there others?

Case 5 n=30. Nothing is known about this problem.

Higher powers

The equation $x^4+y^4+z^4=t^4$. Nothing is known about integer solutions for x,y,z and t.

The equation $x^4+y^4+z^4+t^4=u^4$ probably has infinitely many solutions in positive integers x,y,z,t and u having no common factor. Thus (30,120,274,315,353) Norrie 1911; there are precisely 81 other solutions with u less than or equal to 20469 — what are they? . . . $2^4+2^4+3^4+4^4+4^4=5^4$; $4^4+6^4+8^4+9^4+14^4=15^4$; $1^4+8^4+12^4+32^4+64^4=65^4$.

The equation $(n_1^4-n_2^4)(n_3^4-n_4^4)=m^2$ has solutions (n_1,n_2,n_3,n_4,m)=(3,2,11,2,975) and (2,1,23, 7,2040); are there any others?

Readers are invited to send their attempts at some or all of these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 February 1989. It would be appreciated if such submissions contained a brief description of the programs, details of the hardware used, run times and a summary of results obtained together with suggestions for further investigation in a form suitable for publication.

These submissions will be judged using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Review, June

Suffice it to say that addition chains was not a very popular topic; but extensive references are to be found in RK Guy's book *Unsolved Problems in Number Theory*, page 63 (or SAE to Mike Mudge). However, an outstanding submission was received from Herr M Meuser of Aloysiusstrasse 13,4047, Dormagen 5, West Germany.

The program written in 8080 assembly language was run on a Bondwell Model 2 with Z80 CPU, 50k of free memory and operating system CP/M. The number of addition chains of length 1 for the final value n denoted by ch(n,1) was computed and L(n) deduced from ch(n,1) as the smallest value of 1 for which ch(n,1) is positive.

Herr Meuser verified the results of Knuth, fig 14 at this stage, also conjecturing that $ch(2+1,2n)=(n!)^2$, $ch(2n,2n-1)=n((n-1)!)^2$ and further that $ch(n,n-1)+ch(n+1,n-1)=ch(n+1,n)$.

Sample output from this very worthy prizewinning entry follows in an attempt to encourage further work.

Length	3	4	5	6
Number of chains	7	36	250	2214

Table 1: Number of all chains of a given length

n	3	4	5	6	7	8	l(n)
4	2	·					(2)
5	2	4					3
6	2	8	12				3
7	0	6	24	36			4
8	1	7	37	108	144		3
9	0	3	29	150	432	576	4
10	0	4	37	218	894	2304	4
11	0	0	19	185	1103	?	5
12	:	3	29	248	1614	?	4
13		0	10	157	1452	?	5
14		0	16	204	1875		5
15		0	4	112	1423		5
16		1	13	173	1910		4
17		0	2	68	1184		5
18		0	7	128	1670		5
19		0	0	37	900		6
20		:	6	106	1495		5
21			0	31	755		6
22			0	48	1058		6
23			0	4	416		6
24			4	68	1067		5
25			0	14	371		6
26			0	24	670		6
27			0	5	267		6
28			:	26	620		6
29				0	152		7
30				12	423		6
31				0	80		7
32				20	435		5
33				2	110		6
34				4	201		6
35				0	51		7
36				12	314		6

Table 2: Number of chains of a given length for a given final value

Value	5	6	7	8	9	10
Number	6	22	66	297	1190	6337

Table 3: Number of chains for a given final value.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future articles. All letters will be answered in due course.

Brainteasers courtesy of JJ Clessa. Dec. 88

Merry Xmas to all our readers!

Quickie

Here's a coded message for you to solve. Each letter represents a letter of the original message. The letter 'Q' is unchanged from this original:

'BFS IEZXSA FSG TCIS BFYSB, QIXSZCD BFSA FSG HFXZS ZIZI, WEA QIXZ ZFS TWCCSZ'

Prize Puzzle

This month's puzzle was submitted by Mrs D McClarnon of Swansea who receives our thanks. It's a number crossword for you to mull over while the turkey is cooking. Please send the completed grid, cut out and stuck on a postcard or the back of a sealed envelope, to: December Prize Puzzle, Leisure Lines, *PCW* Editorial, VNU House, 32–34 Broadwick Street, London W1A 2HG to arrive not later than the end of 1988.

Prize Puzzle, September

A moderate response — about 75 entries, from as far afield as Nigeria, Poland, Greece and many other exotic places (Scotland, Yorkshire, Cleveland . . .).

The problem wasn't too difficult, although it did call for programs which could handle large numbers. The answer was 390,903,804 and the winning entry came from Mr R Levy of Glasgow. Congratulations Mr Levy, your prize is on its way.

Clues Across
1: 1d squared.
4: Twice 11d.
5: 5d — 4a.
8: 4a squared.
9: 9d * 10.
10: 26a + 30d.
12: 22a − 6d.
13: 23a + 4a.
14: Quarter of 30d.
17: 11d * 10.
18: 23a − 6d.
19: Same as 20a.
20: Same as 19a.
21: 17a + 7d.
22: Half 38a.
23: 35d + 4a.
25: Same as 17a.
26: Square Root of 40a.
27: Twice 4a.
28: 21a − 31a.
29: 4a + 13d.
30: Same as 26a.
31: 8a − 39a.
33: Twice 21a.
36: 36d − 14a.
38: Twice 22a.
39: Twice 29a.
40: Twice 24d.
41: Same as 11d.
42: 1a * 4.

Clues Down
1: Half of 2d.
2: Square Root of 42a.
3: 17a squared.
4: 4a * 10.
5: 18a squared.
6: 7d * 3.
7: One third of 6d.
9: Square root of 15d.
11: Square root of 17d.
13: 13a + 4a.
14: 18a * 10.
15: 5d * 14a.
16: 7 * 9a.
17: Half of 36d.
18: Half of 22a.
19: 38d * 6.
21: 4a + 10a.
22: 39a + 11d.
24: 20a + 38d.
25: Same as 10a.
26: One less than 35d.
29: 31a squared.
30: Twice 7d.
32: 1a − 16d.
33: 10a + 2d.
34: 27a * 12.
35: 32d * 3.
36: 4a * 11d.
37: 28a squared.
38: 24d − 19a.

Note:
All answers are whole numbers.
28a = the answer to 28 across.
19d = the answer to 19 down.
* = times.
/ = divided by.