

Mike Mudge looks at the generation, storage and analysis of Lucas Sequences.

In the latter half of the last century, E Lucas, working in Paris, considered two distinct one-dimensional arrays which we shall denote by (U_0, U_1, U_2, \dots) and (V_0, V_1, V_2, \dots) . The elements of these arrays are positive integers (or zero) and are defined in two stages:

(i) the first two elements of each array are given, $U_0=0, U_1=1, V_0=2$ and $V_1=P$.
 (ii) Two integer parameters P & Q are specified, and all subsequent elements of the arrays are defined in terms of the two preceding ones by the linear recurrence relations of the second order in terms of P & Q ,

$$U_n = P \times U_{n-1} - Q \times U_{n-2}$$

similarly

$$V_n = P \times V_{n-1} - Q \times V_{n-2}$$

We now illustrate particular examples of these Lucas Sequences.

Example I $P=1, Q=-1$.

Recall that $U_0=0, U_1=1$ and, therefore, using the given values of P & Q in the recurrence relations above:

$$U_2 = U_1 + U_0 = 1, U_3 = U_2 + U_1 = 2$$

$$\text{etc.}$$

$$3, 5, 8, 13, \dots U_{46} = 1836311903 \dots$$

These are the Fibonacci Numbers (see 'Numbers Count', PCW, May 1983).

For $3 \leq n \leq 1000, 21$ Prime Fibonacci Numbers are known, $U_3, U_4, U_5, \dots, U_{569}, U_{571}$. What are the other members of this list? Recall that $V_0=2, V_1=P$ and, therefore, using the given values of P & Q in the recurrence relations above:

$$V_2 = V_1 + V_0 = 3, V_3 = V_2 + V_1 = 4$$

$$\text{etc.}$$

$$7, 11, 29, \dots V_{46} = 4106118243 \dots$$

These are the Lucas Numbers for $0 \leq n \leq 500, 22$ Prime Lucas Numbers are known, $V_0, V_2, V_4, \dots, V_{353}$. What are the

other members of this list? $V_{503}, V_{613}, V_{617}$ and V_{803} are also known to be prime.

Example II $P=3, Q=2$

Exercise for the reader to identify the U_n and the V_n , certainly no computer is needed! Note that $U_{50} = 1125899906842623$ while $V_{50} = 1125899906842625$.

Example III $P=2, Q=-1$. Here it is readily seen that $U_{10} = 2378, U_{20} = 15994428$ and $U_{40} = 723573111879672$. These are the Pell Numbers.

$V_{10} = 6726, V_{20} = 45239074$ and $V_{40} = 2046573816377474$. These are the Companion Pell Numbers.

Example IV $P=4, Q=3$.

$U_{10} = 29524, U_{20} = 1743392200$
 $U_{30} = 102945566047324$
 $V_{10} = 59050, V_{20} = 3486784402$
 $V_{30} = 205891132094650$

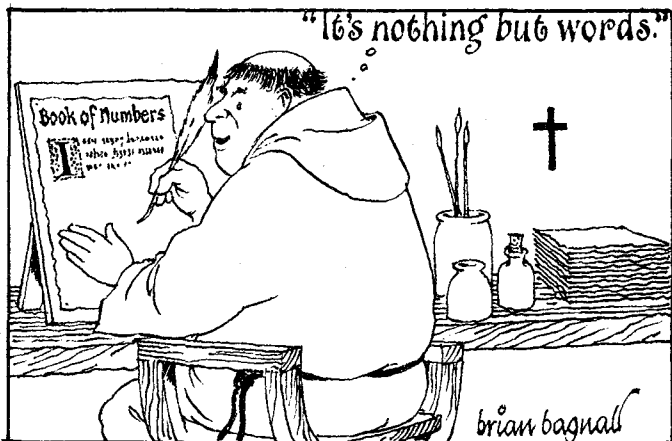
Do these have any particular significance? What are their prime factors?

Detail any other Lucas Sequences which have attracted particular attention in number theory.

Readers are invited to generate Lucas Sequences for particular P & Q values, hence verifying some or all of the numerical results for U_n and V_n given here. Further problems involving the Prime Lucas and Prime Fibonacci numbers, and the identification in Example II and factorisation in Example IV, are hinted at.

Attempts at some or all of these problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South

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10 INPUT "SEED", S 11 IF S > 0 THEN S = S - 1 20 C1 = RND(S)
30 INPUT "MESSAGE", M$ 35 VDU2 40 FOR C1 = 1 TO LEN(M$)
50 C2 = ASC(MID$(M$, C1, 1)) + RND(255) 60 IF C2 < 256 PRINT "";
62 IF C2 < 16 PRINT " "; 70 PRINT " ~C2"; 80 NEXT C1 90 VDU3
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Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 April 1989. These submissions will be judged using suitable subjective criteria and a prize will be awarded by PCW to the 'best' contribution received by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs and run times, and a summary of results obtained, together with suggestions for further investigation; all in a form suitable for publication in PCW.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.
Note Further to the decoding challenge from TK Boyd in PCW, October 1988, the program listing in the box below may help.

Mr Boyd still thinks that without the appropriate seed value a decode is unlikely. Who can prove him wrong?

Review, August

This topic involved Prime Residue Indices and Artin's

constant, it was surprisingly popular, and all interested readers are referred to a book called *Repunits and Repetends* by Samuel Yates, particularly pp102-105.

Mention must be made of the efforts in Forth on an Atari 520 STFM with a single-sided disk drive by Christopher Brooksbank of Peterborough; also the 230 hours of BBC Basic on an Electron attacking Problem B up to 15,000 primes by Frank Webster of Middlesbrough.

However, this month's very worthy prizewinner is Fred Hartley of 46 Hughes Road, Hayes, Middx UB3 3AP, who has used an Archimedes 310 in ARM assembler to calculate indices for 11.6 million primes in about 68 hours. Fred has also carried out a very detailed empirical and theoretical analysis of this problem (details on request) and should be recognised in the literature as and when his results are published.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future articles. All letters will be answered in due course.

PURSUITS

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

Here's one to get you thinking. If the letters of each word in an English dictionary are put into alphabetical order (for example, Fish would become FHIS, Cat would be ACT) and the resulting 'words' were then made into an alphabetical list, which original English word would be the last in the list, and which would be the second? (First would be 'A', of course.)

Prize Puzzle

My thanks to Roy Newham of Nottingham for sending in the idea for this puzzle — almost four years ago. (We may be slow but we got there in the end!)
 A roll of cloth 120 inches wide is cut into a number of lengths. If every length and every diagonal of each piece cut is a different exact number of inches, what is the maximum length that the roll

could be?
 Answers on postcards only (or backs of sealed envelopes) to reach us not later than 28 February 1989. Send your entries to:

February Prize Puzzle,
 Leisure Lines,
 Personal Computer World,
 VNU House,
 32-34 Broadwick Street,
 London W1A 2HG.

Note Winners of the November Prize Puzzle will be announced in next month's issue.

