Square dances

Mike Mudge investigates 'nearly square' primes and follows up a conjecture of Hardy and Littlewood.

This problem began for me with a letter from Dr Charles Lindsay of 96 Princetown Road, Bangor BT20 3TG. Charles defines a 'Trio' to be two prime numbers and the perfect square with respect to which they are symmetrically placed. Algebraically we can write a Trio as $T=(p_1,n^2,p_2)$ where $n^2-p_1=p_2-n^2=w$, (width of Trio).

A simple investigation reveals Trios thus: (7,9,11); (61,64,67); (79,81,83); . . . (1669, 1681, 1693); (2593, 2601, 2609); . . .

Problem 1 Is there a formula for the n^{th} Trio?

Problem 2 Are there infinitely many Trios?

An attempt to discover the background to this problem was a failure, however it revealed what appears to be a related problem from Hardy and Littlewood, 1923. These most eminent mathematicians conjectured that:

'There exist infinitely many prime pairs of the form (m^2+1,m^2+3). The number of such primes m^2+3 less than n is given asymptotically by:

Q(n) approaching

$$\frac{3(n)^{1/2}}{(\log n)^2} P\left(\frac{p(p-v)}{(p-1)^2}\right)$$

where we use P to denote the product of all terms with p greater than 3, and v is defined by:

$$v=\begin{cases} 0 \text{ when } (-1/p)=(-3/p)=-1 \\ 2 \text{ when } (-1/p)(-3/p)=-1 \\ 4 \text{ when } (-1/p)=(-3/p)=+1.\text{'} \end{cases}$$

Mathematically inclined readers should note that (n/p) is the Legendre Symbol defined as +1 if n is a quadratic residue modulo p and −1 otherwise.

Problem 3 Construct the sequence of prime pairs of the form (m^2+1,m^2+3).

Note These are simply Trios of width 2.

Problem 4 (Mathematics permitting.) Evaluate Q(n) for various n (that is, the upper limit to the search for Trios of width 2) and provide empirical evidence for the asymptotic distribution conjectured above.

Alternative Problem 4 What happens to Charles Lindsay's original concept if squares are replaced by cubes, fourth powers, fifth powers and so on.

In other words, are there generalised trios ('GTs') given by (p_1, n^r, p_2) having width defined by $n^r-p_1=p_2-n^r={}^wGT$?

Attempts at some or all of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 June 1989. Any submissions received will be judged using suitable subjective criteria and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, October

This problem was concerned with the evaluation of a quadratic polynomial with integer coefficients for a sequence of integer argument. Basic question: for how long can the results have modulus unity or be prime? How many of the results are found to have this property if the range is extended beyond the initial one? V(f(x),N) was defined on page 240, *PCW*, October 1988.

The prizewinner this month is Bruce Halsey of 31 Marlborough Green Crescent, Martham, Great Yarmouth, Norfolk NR29 4ST.

Bruce used Fast Basic on an Atari 520ST and offers the following empirical result: 'On looking at my results I think that the highest values of V should be produced by quadratics where a=1, b is an odd number of about −1000 and c odd around 125,000 . . . I would be glad to share my sheaf of results with anyone else interested in the subject.' Take up this offer and let us hear more about V(f)x),N)!

Robin Merson, a regular correspondent to this column, has examined in detail the case N=1000 and finds the optimal solution of 659 primes for (a,b,c) = (1,35,248063) and a sub-optimal solution of 657 primes for (a,b,c)=(1,1,247757).

Reg Bond, another 'Numbers Count' stalwart, has recollected his computations of between 1½ and 10 years ago to yield estimation formulae giving approximations to the number of prime values of x^2+x+p for $1 \leq x \leq N$; details on request.

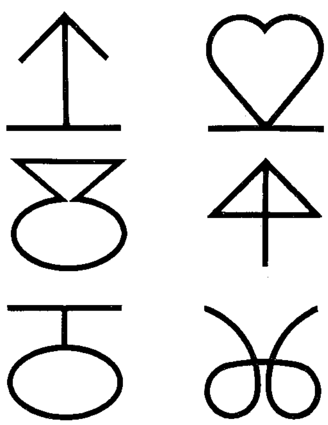
Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

This month's quickie has been supplied by Master Richard Sparnon of Morecambe who challenges you to find the next member of the following series:



Many thanks, Richard – that should keep a few people guessing!

Prize Puzzle

This month's problem is one of logic, but for those who prefer to use sledgehammer methods, it can be done by micro.

In the following multiplication, each letter represents a digit. What digit is Q?

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S P R I N T
          Q
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P R I N T S
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Answers on postcards or backs of envelopes to: *PCW* Prize Puzzle April, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 April 1989.

Prize Puzzle, January

A very low response to this problem – only 14 entries were received. It wasn't really a true problem for micro solution, but we did have an outright winner and the random number generator was not needed.

The winning entry came from Mike Waterman of Camberley who receives our congratulations. Unfortunately, Mr Waterman, we could not decipher your address. If you care to drop me a line, you will receive your prize.

Mr Waterman's solution requires only 16 sevens:

7 ✱ [7! +77 +7·7 +7·7] ✱ √(·7 + ·7⁷) +7! −77 −7/ ·7