PURSUITS

Sequential processing

Mike Mudge invites non-specialist mathematicians to solve straightforward problems, although, courtesy of Douglas Hofstadter, they may not be so simple.

This month's column is a response to many letters I have received, which are typified by: 'your Numbers articles in *PCW* have given me much pleasure over the years, although many of them have been beyond my scope!'

There is no requirement for mathematical training beyond that of second-form algebra in what follows — although the scope for ingenuity of programming, leading to the discovery of new results, is considerable.

Please read on!

Sequence A

a_1, a_2, a_3, a_4 . . . is a non-terminating sequence of positive integers defined as follows:
$a_1=a_2=1$, $a_n=a_{n-a_{n-1}}+a_{n-a_{n-2}}$ for n greater than 2.

Thus $a_1=1$, $a_2=1$, $a_2=a_{2-1}+a_{2-1}=1+1=2$, $a_3=a_{3-2}+a_{3-1}=2+1=3$, $a_4=a_{4-3}+a_{4-2}=1+2=3$ and the sequence continues 4,5,5,6,6,6,8,8,8,10,9,10,11,11, 12,12,12,12,16,14,14,16,16,16, 16,20,17,17. . .

Problem A Write a computer program to generate this sequence. Do *not* simply list all of the terms computed, but rather address the question: 'Are there infinitely many integers (7,13,15,18. . .) that are missed out?'

Of course a finite computer program cannot provide a yes/no answer to this question, but it will be interesting to see a list of such 'missed out' numbers up to some large value.

Sequence B

b_1, b_2, b_3, b_4. . . is a non-terminating sequence of positive integers defined as follows:
$b_1=1$, $b_2=2$, and for n greater than two, b_n is the least integer greater than b_{n-1} which can be expressed as the sum of two or more *consecutive* terms of the sequence.

Thus the sequence begins: 1,2,3,5,6,8,10,11,14,16,17,18,19, 21,22,24,25,29,30,32,33,34,35, 37,40,41,43,45,46,47. . . where, for example, the last term quoted (namely 47) is the least integer greater than the previous term (46) and can be expressed as 1+2+3+5+6+8+ 10+11. In this case it is the sum of eight consecutive terms of the sequence.

Note that, when determining the sixth term of the sequence, having already obtained 1,2,3,5,6 it is not found to be possible to construct 7 as the sum of two or more consecutive terms and so 3+5=8 yields the next term.

Problem B Write a computer program to generate this sequence. Again study those integers which are missed out.

Sequence C

c_1, c_2, c_3, c_4. . . is a non-terminating sequence of positive integers defined as follows:
$c_1=2$, $c_2=3$ and when c_1. . .c_n are defined, form all possible expressions c;c;−1 where $1 \leq i < j \leq n$ and append them to the sequence, thus obtaining: 2,3,5,9,14,17,26,27,33,41,44,50, 51,53,69,77,80,81,84,87,98,99, 101,105,122,125,129. . .

Notice that to 2,3, we append $2\times3-1=5$; to 2,3,5, we append $2\times5-1=9$ and $3\times5-1=14$, and so on.

Problem C Write a computer program to generate this sequence. Attempt to estimate what fraction of the integers less than 10^n appears in this sequence, for n=1,2,3,4. . .

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF; tel: (0902) 892141, to arrive by 1 August 1989.

Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, December

The subject of Diophantine equations has occupied the attention of mathematicians over a period of many centuries, starting with the Greeks, Arabs and Indians. Readers requiring an account of the most interesting and important results are encouraged firstly to consult *Diophantine Equations* by LJ Mordeli (Academic Press, 1969).

More recent results are distributed throughout the literature of both pure and computational mathematics and can be most efficiently explored using *Mathematical Abstracts* or *The Science Citation Index*.

'Take three cubes': Case 2 has no further solutions satisfying Mod(x+y+z) less than or equal to 150000. Case 5 trivially has no solutions in positive integers. Proof?

Now to $x^4+y^4+z^4=t^4$. Noam D Elkies (*Mathematics of Computation*, volume 51, number 184, October 1988, pages 828-835) initially discovered the solution (2682440, 15365639, 18796760, 20615673) by a direct computer search technique, followed by a second independent solution with digits of the order of 10^{70} *not* by direct search!

However, Noam's work led Roger Frye of Thinking Machines Corporation to find the minimal solution (95800, 217519, 414560, 422481) as reported by Keith Devlin, *Computer Guardian*, May 1988.

This month's prizewinner, for interest in and literature relating to Diophantine equations, is Reg Bond of 75 Laburnum Crescent, Allestree, Derby DE3 2GS.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general of specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

No prizes — not even an answer for this one. A boy cycles to school at 10mph and returns home at 15mph. What is his average speed for the complete round trip?

Prize Puzzle

Now here's a problem that should please any 'Greens' among our readers. It's about an orchard which originally contained 100 trees set out symmetrically as a 10x10 grid at 10-yard intervals between trees horizontally and vertically.

Each tree was numbered as follows: trees 1–10 were in the first row; trees 11–20 were in the 2nd row; trees 21–30 were in the 3rd row, and so on.

One night a storm arose and, by coincidence, uprooted every tree that was numbered with a prime number — that is, trees numbered 1, 2, 3, 5, 7, 11 and so on — leaving only 74 standing.

Now for the problem. In this depleted orchard, how many different ways can you find four trees which form the corners of an exact square, of any size?

That should get the micros whirring (or the wellies out, for those who prefer to do it manually).

Answers on postcards or backs of envelopes to: Leisure Lines Prize Puzzle June 1989, *Personal Computer World* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive no later than 30 June 1989.

Winner, Prize Puzzle, March

A more-difficult-than-usual problem; some 80-odd entries were received with a few incorrect solutions. The answer:

33 577 577 777 777 775

The winning entry — drawn, as usual, at random from the correct entries — came from Bonnie Scotland, from Mr Graeme Hughes of Glasgow. Congratulations, Graeme — your prize is on its way.

Meanwhile, to all the also-rans, keep puzzling — it could be your turn next!