# Factorial functions

**A set of five problems involving the factorial function, presented by Mike Mudge.**

**Definition** Given a positive integer, n, the function FACTORIAL n is defined as the product of all of the positive integers up to and including n. We write FACTORIAL n as n! thus: n! = 1x2x3x4x . . . xn. For example 1! = 1, 7! = 1x2x3x4x5x6x7 = 5040.

**Problem 1** When is n! expressible as the sum, or difference, of integer powers of a given integer? that is n! = $m^a \pm m^b$. Erdös observed that n! = $2^a \pm 2^b$ only when n = 1,2,3,4,5. For example 5! = 120 = 128 − 8 = $2^7 - 2^3$.

**Problem 2** When is n! expressible as a product of three consecutive integers? That is, n! = (m − 1)(m)(m + 1). Simmons, *Journal of Recreational Mathematics,* vol 1, 1968, p38, found four solutions (m,n) = (2,3), (3,4), (5,5) & (9,6). For example 6! = 720 = 8x9x10.

**Problem 3** When is n! + 1 a prime number? That is, n! + 1 = p where p is divisible only by itself and unity. Templer, *Mathematics of Computation,* vol 34, 1980, pp303-304, found eleven solutions: n = 1,2,3,11, . . . 154.

**Problem 4** When is n! expressible as a product of two or more non-trivial factorials. That is, n! $a_1!\mathrm{x}a_2!\mathrm{x}a_3!\mathrm{x} \ldots a_r!$ where r and each $a_s$ are greater than one. Hickerson observed:

9! = 7! x 3! x 3! x 2!
10! = 7! x 6! = 7! x 5! x 3!
16! = 14! x 5! x 2!

**Problem 5** How close to n! do the integer powers of a given integer become? That is given an integer r find the power m which minimises the modulus of n! − $r^m$.

Croft computed a table for the case r = 2 including:

| | | | |
|---|---|---|---|
| n | 5 | 20 . . . . . . . . . | 90 |
| m | 7 | 61 . . . . . . . . | 459 |
| d | −1.34 | +1.26 | −0.0007 |

Where d is the *percentage* error in the *exponent* m.

Can d be further reduced for a given (or all) n by increasing r from 2 through the values 3,5, . . . etc?

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF. tel: (0902) 892141, to arrive by 1 September 1989. Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication and a sae.

## Review, Jan 1989

This problem related to the factorisation of Binomial Coefficients, a topic first discussed in 'Numbers Count', *PCW* October 1984. A very readable early reference is the paper of P Erdös and G Szekeres, 'Some number theoretic problems on binomial coefficients', *Australian Mathematical Society Gazette,* vol 5, 1978, pp97-99.

However, the most recent paper known to the writer is that of Pierre Goetgheluck, 'On prime divisors of binomial coefficients,' *Mathematics of Computation,* vol 51, no 183, July 1988, pp325-29; which, 'using computational and theoretical methods, deals with prime divisors of binomial coefficients: geometric distribution and number of distinct prime divisors are studied. We give a numerical result on a conjecture by Erdös on square divisors of binomial coefficients.'

Other references which may be relevant include:
A Sarközy, 'On divisors of binomial coefficients I; *Journal of Number Theory,* vol 20, 1985, pp 70-80.
PAB Pleasants, 'The number of prime factors of binomial coefficients'; *Journal of Number Theory,* vol 15, 1982, pp203-225.
P Erdös, H Gupta & SP Khare, 'On the number of distinct prime divisors of $\binom{n}{k}$'; *Utilitas Math* vol 10, 1976, pp51-60.

Worthy responses include:
**(1)** Michael J Cowan using Basic/Assembler on an Apple IIe with twin disk drives. Michael attacked problem 3* and identified a shortage of memory rather than time: his calculation of w(2n,n) in the case of n = 30000 taking 2.43 hours.
**(2)** Mathias Meuser used 8080 assembly language on a Bondwell 2 with 2MHz Z80 CPU and 50k of free memory running under CP/M. He considered problems 1, 3* & 4 determining that w(1000,353) = 109 with a largest prime factor of 997 in 25 seconds, while w(30000,10000) = 2039 with a largest prime factor of 29989 was established in 30 minutes.

However, after much thought, this month's worthy prize winner is John Cannell, of 28 The Ridge, Surbiton, Surrey KT5 8HX.

All John's programs are in interpreted Basic running on a BBC Master; a measure of speed being 1.05 seconds for the expansion into prime factors of $\binom{1000}{353}$ compared with 'less than half a second' by Goetgheluck using compiled Pascal.

A feature of John's submission was the graphical display of **(a)** Distinct Prime Factors of $\binom{n}{r}$ and **(b)** Powers of Prime Factors of $\binom{n}{r}$. John showed familiarity with Goetgheluck's work and suggests an alternative to E(n) = n1n(4)/ 1n(n), due to Erdös as a limiting form for w(2n,n) as n becomes very large; that alternative being C(n) = 1.358n/(1n(n) − 0.377).

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general of specific, for future Numbers Count articles. All letters will be answered in due course.

# LEISURE LINES

**Brainteasers courtesy of JJ Clessa.**

**Quickie**

No answers, no prizes and no apologies from us for this hoary old chestnut:

Three men order a meal in a restaurant. The bill comes to £30, so each man puts down a £10 note as his share. The waiter takes the money with the bill over to the cash desk where the manager has a twinge of conscience and decides he's overcharged the men, so he knocks £5 off the bill — making it £25 for the three meals. He tells the waiter to give the men £5 back. Now the waiter, seeing some easy money, decides to charge the men £27, keeping £2 for himself. So what has happened to the other pound?

**Prize Puzzle**

This month's problem has been submitted by Anthony Isaacs of London. Many thanks, Mr Isaacs — let's hope the answer you gave me is the correct one!

Using all the digits 0-9 (but no leading zeros), what is the smallest number that can be made which is exactly divisible by every number from 0-18?

Answer on postcards (or backs of sealed envelopes) to: Leisure Lines, Prize Puzzle, July 1989, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG. It should arrive no later than 1 August 1989.

Good Luck!

**Prize Puzzle, April**

A very simple problem indeed — 165 entries received — but nevertheless, quite a few of these actually had the wrong answer.

The correct answer is Q=3, and the winning entry, drawn at random from the heap, came from Mark Datko of Luton. Congratulations Mr Datko, your prize is on its way.

Meanwhile, to all the alsorans, keep trying — it could be your turn next. This month's puzzle could be your path to fame, glory and riches.

Well, perhaps not quite that, but you could win yourself a beautiful Faber-Castell stainless steel automatic pencil at least!

By the way, we apologise for a misprint in the solution to the January puzzle. The correct answer using 16 7s should have been:

13579.02468 =
$7 * [7! + 77 + 7.7 + 7.7] * \sqrt{(.7 * .7^7)} + 7! - 77 - 7/.7$

Sorry about that, and once again let me wish you good luck with this month's problems.