# The Maltese Factor

## Mike Mudge considers partitions into sums of consecutive integers.

| Number of ways | Smallest integer | Prime factors |
|---|---|---|
| 1 | 3 | 3 |
| 2 | 9 = 5+4 | 3**2 |
| | = 4+3+2 | |
| 3 | 15 = 8+7 | 3*5 |
| | = 6+5+4 | |
| | = 1+2+3+4+5 | |
| 23 | 3465 = ? | 3**2*5*7*11 |

This problem has been suggested for 'Numbers Count' by Mr Albert Debono, a regular reader of this column living in Malta.

The proposal is to investigate the number of ways in which a given positive integer can be expressed as the sum of consecutive positive integers.
For example:

$$95 = 47 + 48 = 17 + 18 + 19 + 20 + 21$$
$$= 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14.$$

**Problem 1** To construct a computer program which, given a positive integer, N, as input would output all the various ways in which N can be written as the sum of consecutive positive integers.

Albert conjectures that the only numbers which will not be expressible in this form are the integer powers of two, that is 2, 4, 8, 16, . . .

**Problem 2** To determine the smallest integer which can be expressed as the sum of consecutive positive integers in a given number of ways, and to express these integers in their prime factors. Thus to obtain a table, see the box above.

Investigation of this table suggests the following:
3 is always a factor of the smallest integer that can be expressed as a sum of consecutive integers in n ways? In general, even n generate far larger smallest N than do odd n?

Are these suggestions valid, and if so why?

**Problem 3** Extend the above ideas to the representation of a given positive integer as the sum of *consecutive prime numbers*. For example,

$$100 = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23$$
$$= 47 + 53.$$

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 October 1989. Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

## Numbers Count Review, February 1989

Among the numerical results requested are:
**Prime Fibonacci Numbers** $U_n$ occur when n = 3,4,5,7,11,13,17,23,29,43,47,83, 131,137,359,431,433,449,509, 569,571. .2971. . .?

**Prime Lucas Numbers** $V_n$ occur when n = 0,2,4,5,7,8,11,13,16,17,19,31,37, 41,47,53,61,71,79,113,313,353, . .503. .613. .617. . .803. . .?
Example II Answer $2^n \pm 1$.

A full discussion of this topic is to be found in Paulo Ribenboim's *The Book of Prime Number Records*, Springer-Verlag 1988.

No submission received justifies the award of a prize this month; thus it is proposed to re-open the topic of Lucas Sequences until 1 October 1989 so there is a second chance to submit any relevant material.

> Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

## Brainteasers courtesy of JJ Clessa.

**Quickie**
No answers, no prizes.
Another chestnut — but can you solve it? A brick weighs seven pounds plus half a brick. What is the weight of a brick and a half?

**Prize Puzzle**
A slightly different problem this month, but one that could get the micros whirring. You are given nine digits and a blank grid. You have to fit the digits so that the clues are matched.

**Digits to be used**

1 3 3 3 5 7 8 8 9

**Clues Across**
a Multiple of a perfect square
d Digits in arithmetical progression
e Exact multiple of 11

**Clues Down**
a Palindromic number
b Digits add up to 17
c Prime number

Answers on postcards or backs of envelopes only, please, to arrive by 1 September 1989.

Send your entries to: Leisure Lines Prize Puzzle August, PCW Editorial Office, VNU House, 32-34 Broadwick Street, London W1A 2HG.

**Winner May 1989**
A smaller than average entry. Exactly 98 entries were received, and several of these had the wrong answer.
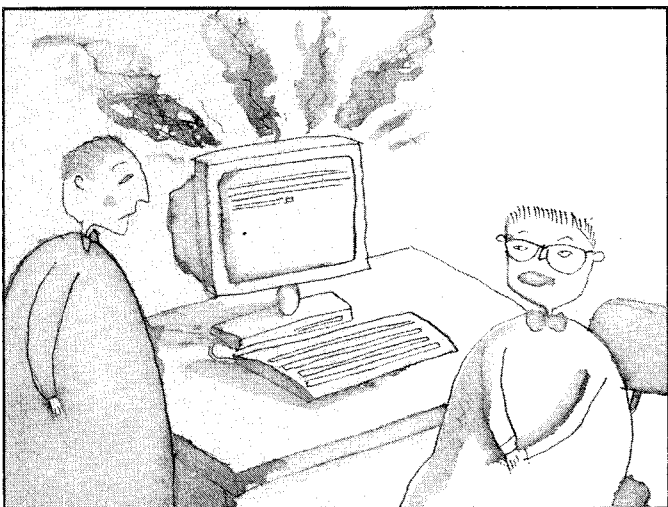
'It says it's lost its entire program and can it have a Hamlet cigar.'

The problem is relatively easy if a computer is used, and the required answer is 19019 which is the product of the four prime numbers 7, 11, 13 and 19.

The first correct winning entry, drawn at random from the pile, came from a member of the services — Ms Rhona Knudsen, at BFPO 50 (wherever that is). Congratulations, Rhona, your prize is on its way.

Meanwhile, to the rest of you, keep trying — this month's puzzle could prove to be your turn to win a luxurious Faber-Castell stainless steel automatic pencil.