NUMBERS COUNT

Mike Mudge investigates Goldbach's Conjecture or 'How can something so simple be so difficult'.

In 1742 the Russian mathematician Christian Goldbach (1690 — 1764) wrote to the Swiss mathematician Leonhard Euler (1707 — 1783) expressing a belief that: 'Every integer greater than 5 is the sum of three primes.' Euler replied that this is easily seen to be equivalent to: 'Every even integer greater than 4 is the sum of two primes.'

Proof left as an exercise for the reader, or see page 229 of *The Book of Prime Number Records* by Paulo Ribenboim (Springer Verlag, 1988).

In 1937 the Russian mathematician IM Vinogradov gave a simplified proof of an earlier result due to GH Hardy and JE Littlewood, 1923, namely that there exists an integer n_0 such that every odd number greater than or equal to n_0 is the sum of three primes. One calculation of n_0 yields ${}_3 3^{15}$, quite a large number!

Here we leave the theoretical results and examine some empirical results whose reproduction and extension are well within the scope of a typical personal computer.

Problem 1 In his recent book, *Invitation to Number Theory with Pascal* (Camelot Publishing Company, 1989), Donald D Spencer defines a Silverbach Number as an integer that can be expressed as the sum of two primes in three different ways. Thus: 22 = 19 + 3 = 17 + 5 = 11 + 11.
Write a program to generate such numbers, and examine their possible asymptotic frequency.

Problem 2 Spencer also defines a Copperbach Number as an integer that can be expressed as the sum of two primes in two different ways. 18 = 5 + 13 = 7 + 11; 20 = 17 + 3 = 13 + 7
Write a program to generate such numbers, and examine their possible asymptotic frequency.

Problem 3 The alternative title to this month's column is taken from the fascinating book *A Number for Your Thoughts* by Malcolm E Lines, (Adam Hilger, 1986) where a related decomposition is discussed.

Twin Primes are defined to be two prime numbers differing by 2, thus 3, 5; 5,7; 11,13; 17,19; 29,31; 41,43; . . . see *PCW*, July 1984 for a related computational problem discussed in detail in an article entitled 'Brun's Constant' by Ed Rosenstiel.

Lines reports that a computer search of all even integers less than one million found that those greater than 4208 could be represented by the sum of two primes taken from the sequence of twin primes; further, that all even integers larger than 24,098 and less than a million could be expressed as the sum of two twin primes in more than one way, 'Some in more than one thousand different ways.'

Verify the results of Lines and tabulate the number of possible decompositions into the sum of two twin primes for all integers less than one million.

Problem 4 Returning to the Goldbach decomposition into the sum of two primes (for even integers), the question to be addressed is: how to determine the number N above which there are at least n different Goldbach decompositions.

Clearly, the computer search can only examine a finite number of values greater than N but suggested results are in the table below.
Verify these results and generate additional values for the table. In how many ways can 1000000 be decomposed into Goldbach Pairs?

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 November 1989.

Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained, together with suggestions for further work; all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

n	50	100	150	600	650	700
N	4688	11672	19246	105368	116618	126878

Suggested results for Problem 4

Review, March

The Chinese Remainder theorem has its earliest known formulation in the Sun Tzu Suan-ching (that is, the mathematical classic of Sun Tzu) which has been dated between 280 AD and 473 AD. In modern times it has formed the basis of 'mind reading' tricks both as party pieces and on the stage. However, readers were asked to consider its application to multi-length, computer-oriented arithmetic.

Several complete implementations were received, together with considerable scepticism regarding its value as a computing tool. Sceptics see D Knuth, *The Art of Computer Programming: Semi-Numerical Algorithms, Volume 2* (2nd edition, Addison-Wesley 1981).

This month's worthy prizewinner is Ed Hersom of Glen Cottage, Bagby, Thirsk, N Yorks YO7 2PF, using Forth on his system which 'has an 80186 chip and an 8087'. Ed gained much inspiration from *Microchip Mathematics — number theory for computer users* by Keith Devlin, which I strongly recommend to all interested readers.

Ed removed the need for the addition of two 'long numbers', which is troublesome, and required only their input, output and the determination of their relevant residues. Altogether a very worthwhile piece of work.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa. Sep 89

Correction, July 1989
Whoops, we goofed! Our apologies to readers for a mistake in the July Prize Puzzle. The problem was stated as follows:

Using all the digits 0-9 (but no leading zeros), what is the smallest number that can be made which is exactly divisible by every number from 0-18?

Clearly the range of divisors of 0-18 is a nonsense, and should have read 1-18.

This month's quickie
No answers, no prizes. You are probably aware of English words in which the vowels a, e, i, o, and u appear in sequence — the word 'facetious' is one of the best known. Can you find a word in which the vowels appear in reverse sequence — that is, u, o, i, e, and a?

Prize Puzzle
Regular puzzlers will, no doubt, be aware of the problem of the cheque in which the pounds and the pence figures are transposed. The standard problem goes something like this:

A lady receives a dividend cheque from her bank manager in which the pounds and the pence values have been transposed. However, unaware of this, she cashes the cheque and spends some of the money (X) before she realises the mistake. She then calculates that at this stage the money she has remaining is an exact multiple (N) of the amount that the cheque should have contained. What should this original cheque have been?

The classic problem then varies depending upon the values of the amount spent (X) and the multiple (N).

In this month's problem, we have six ladies and six transposed cheques. Each lady spends the same sum (X) as the others, and each finds, as before, that she is left with an exact multiple of the intended amount. However, each of these multiples (N) is different. What is the amount spent (X) in each case?

Answers on postcards or backs of sealed envelopes only, to Prize Puzzle September 1989, PCW Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive before 1 October 1989. END