NUMBERS COUNT

A little studied constant due to A Ya Khninchin, presented by Mike Mudge.

Some algebraic notation and definitions:

Repeated Summation

$$\sum_{r=1}^{n} a_r = a_1 + a_2 + a_3 + \ldots + a_n$$

$$\sum_{r=1}^{\infty} a_r = a_1 + a_2 + \ldots$$

For example,

$$\sum_{r=1}^{3} 1/r = 1/1 + 1/2 + 1/3 = 11/6$$

$$\sum_{r=1}^{\infty} 1/2^r = 1/2 + 1/4 + 1/8 + 1/16 \ldots = 1$$

8/9 *(geometric series)*

Repeated Multiplication

$$\prod_{r=1}^{n} a_r = a_1 \times a_2 \times a_3 \times \ldots \times a_n$$

$$\prod_{r=1}^{\infty} a_r = a_1 \times a_2 \times \ldots$$

For example

$$\prod_{r=1}^{3} 1/r = 1/1 \times 1/2 \times 1/3 = 1/6$$

$$\prod_{r=1}^{\infty} 1/r = 1/1 \times 1/2 \times 1/3 \times \ldots = 0$$

(Numerator remains 1, while the denominator becomes larger and larger.)

Natural Logarithms
The natural logarithm of a equals b, written 1n a=b, means that b is the power to which the base, e (approx 2.718281828) must be raised to produce b. For example, 1n 1.284 is nearly 0.25=1/4 because (2.718281828)¼; that is, the fourth root of 2.718281828 is nearly 1.284.

The arithmetic mean, A (average) of n quantities, $Q:(a_1, a_2, \ldots a_n)$ is defined by $A=(1/n)(a_1+a_2+a_3+\ldots a_n)$.

The geometric mean, G (average) of the same n quantities is defined by $G=n\sqrt{(a_1 x a_2 x a_3 \ldots x a_n)}$.

For example, if Q:(1,2,3,4,5) then A = (1+2+3+4+5)/5=3 while G=5√(1×2×3×4×5) approx 2.605.

A continued fraction expansion
Given any positive number, x, the associated continued fraction expansion, $c_0; c_1, c_2, c_3, \ldots$ is defined as follows:

c_0 is the whole number part of x, c_1 is the whole number part of $1/(x-c_0)$ and so on, thus x=

$$c_0 + \cfrac{1}{c_1 + \cfrac{1}{c_2 + \cfrac{1}{c_3 + \ldots}}}$$

We write $x=(c_0; c_1, c_2, c_3, \ldots)$ thus a simple calculation shows that 105/38=(2;1,3,4,2).

Definition, Khinchin's Constant, K,

$$K = \prod_{r=1}^{\infty} (1+1/(r^2+2r))^{\frac{\ln r}{\ln 2}}$$

Note carefully that this is the *product* of infinitely many terms, each of which consists of the simple expression $(1+1/(r^2+2r))$ raised to the *power*(1n r)/(1n 2).

Problem (1) Compute, K, to as many *correct* significant figures as possible. Clue — K is approximately 2.6......

Problem (2) Determine the continued fraction expansion of the approximation to, K, which has been calculated in (1).

Warning do not allow the continued fraction algorithm to continue beyond the point where significant figures in, K, are no longer correct.

Problem (3) Investigate the postulate that the geometric mean of the terms in the continued fraction expansion of, K, (neglecting $c_0=2$; that is, the whole number part of, K,) approaches, K, as more of them are included.

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel (0902) 892141, to arrive by 1 December 1989. Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, April 1989

The problem related to TRIOS and the proposer, Charles Lindsay, provoked only limited response. Unfortunately, the definition given of a TRIO referred to 'two prime numbers' when 'two *consecutive* prime numbers' was intended. However, this point was appreciated by most readers.

Thanks are due to Glenn Taylor for explaining that Problem 2 and alternative Problem 4 are indeed subsets of The Goldbach Conjecture.

Charles Lindsay has broadened the issue with the observations that:
(1) Several TRIOs lie on the same arithmetic progression, so does every (suitable) arithmetic progression contain a (finite?) number of TRIOs?
(2) When an arithmetic progression ceases to have squares mid-way between consecutive primes, do they continue to appear between non-consecutive primes?
(3) It seems likely that all squares are the arithmetic mean of two primes, which are not usually consecutive, so that TRIOs are in some sense 'the tip of an iceberg'.

Comments from readers to myself or Charles directly. This month's prizewinner is John Sutton of 8 Porchester Street, Ascot, Berkshire SL5 9DY. John used his BBC Master to fill (almost) a large ADFS disk. This data was then subjected to various modes of graphical analysis, although no strong conjectures were forthcoming. 'The constant of proportionality in the limit expression for Q seems to lie between 5.5 and 5.7.' Can anyone improve on this?

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie
No prizes, no answers, for this one. A girl has as many sisters as she has brothers, but each brother has twice as many sisters as he has brothers. How many brothers and sisters are there in the family?

Winner, July 1989
Probably a bit too easy. There were just over 100 entries, but 20 of these got it wrong (and not because of our goof, which we owned up to in the last issue).

The solution, the smallest number containing all the digits 0-9 which is exactly divisible by every integer from 1-18, is:

2, 438, 195, 760

The winning card drawn at random from the correct entries came from Ms Kathryn Wyatt of Aberdare, Wales, who receives our congratulations and a beautiful prize which will be coming her way shortly.

Prize Puzzle, October 1989
Here's a problem that shouldn't cause you too much grief. Each of the five symbols used in the grid alongside represents a different integer number. However, one of these symbols has been omitted from one square. Using the row and column totals shown, can you deduce which symbol the blank square should contain?

	#	$	$	63
#	$	@	@	50
$	*	&	*	63
#	&	@	#	48
59	61	45	59	

Answers on postcards or backs of sealed envelopes only, to arrive not later than 31 October 1989, to: October Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG.