Mike Mudge investigates Greedy Sequences and The Least Integer Solution of the Diophantine Equations.

This subject area has been suggested by a regular 'Numbers Count' correspondent, Peter Cameron of Oxford, and all responses received will be forwarded to him for information.

Definition A 'greedy sequence' is a sequence of positive integers obtained from the natural numbers 1, 2, 3, 4, 5, . . . by imposing a condition to *forbid* certain numbers.

Example A The greedy sum-free sequence contains no three terms s, t, u with $s + t = u$, where s & t may be equal. Thus 1 is in, 2 is out since $1 + 1 = 2$, 3 is in, 4 is out since $1 + 3 = 4$, and so on. The sequence of odd numbers is generated.

Example B The greedy sequence in which it is forbidden for any term to be a factor of any other term. 1 must be excluded from the initial sequence or everything else is forbidden! 2 removes all even numbers, 3 removes all multiples of 3, and so on, and the sequence of prime numbers is generated.

Problem (i) The Sidon sequence arises by forbidding $x+y = z+w$ for any four distinct terms x, y, z & w. This initially looks like the Fibonacci sequence (*PCW*, May 1983) but how does it continue? *What is the 10^rth term for r = 1, 2 . . . and about how large is the n^{th} term?*

Problem (ii) The MacMahon sequence arises by forbidding any term that is the sum of terms in a subsequence of consecutive terms. This begins 1, 2, 4, 5, 8, 10, . . . but how does it continue? Give a geometrical interpretation of this sequence. Consider question *. . .* above.

Problem (iii) The sequence in which the sum of two distinct terms never divides their product. This sequence, which initially includes most numbers (note, 6 is out since 3+6 divides 3×6) but appears to become less dense, shall be known as the Cameron sequence. Consider question *. . .* above.

Problem (iv) Compute the first 10^r terms for r= 1, 2, 3 . . . and hence, or otherwise, find the patterns for the greedy sequences defined by: **a)** no three terms shall be in arithmetic progression — that is, no three terms p, q & r shall satisfy $q - p = r - q$. **b)** no three terms p,q & r shall satisfy $p \times q = r$ — that is, product-free.

The Least Integer Solution of the Diophantine Equations: $s = x^3+y^3 = z^3+w^3 = u^3+v^3 = m^3+n^3$.

This has been computed by E Rosenthal, J A Dardis and C R Rosenstiel using a personal computer and they have proved that $6963472309248 = 2421^3 + 19083^3 = 5436^3 + 18948^3 = 10200^3 + 18072^3 = 13322^3 + 16630^3$ is the least positive integer which can be represented as the sum of two cubes in *four* different ways.

The least positive integer which can be represented as the sum of two cubes in *three* different ways was found by Leech J, *Trans Camb Phil Soc* v53, part 3, July 1957, pp778-780. $87539319 = 167^3 + 436^3 = 228^3 + 423^3 = 255^3 + 414^3$ and is listed in Wells D, *The Penguin Dictionary of Curious and Interesting Numbers*, London, Penguin Books, 1988, p189.

The least positive integer which can be represented as the sum of two cubes in just *two* different ways is $1729 = 9^3 + 10^3 = 1^3 + 12^3$ which is of course the substance of the tale regarding Ramanujan's taxi. See for example Rankin R A, *IMA Bulletin*, 1987, v23, p149.

Attempts at some, or all, of the above problems, together with comments upon or extensions to the result of E Rosentiel *et al*, may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel (0902) 892141 to arrive by 1 January 1990. Solutions to the problems received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained, together with suggestions for further work, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Winner, August 1989

Another not too difficult problem — or at least, so it seemed to almost 200 entrants. However, about 25% got the wrong anser. One letter from overseas consisted of three pages of logic proving why the problem had no solution!

Many people didn't realise that digits in arithmetical progression could be in descing as well as ascending order. The correct solution — which is unique — is:

a 8	b 3	c 3
d 7	5	3
e 8	9	1

The winning card, drawn as usual randomly from the correct entires, came from Scotland — Mr Neil Jarvis of Edinburgh. Well done, Neil — your prize is on its way.

Meanwhile, to the also-rans — keep trying, it could be your turn next. But a reminder: don't send your solutions in letters — we have to disqualify them. Postcards or backs of sealed envelopes only, please. Also remember to tell us your name and address — there's always one or two entrants who forget to do this. There have been a few occasions on which the card drawn at random had no name and address on it, so someone missed a prize. Don't let it be you.

Prize Puzzle

This month's problem is a pathfinder puzzle. It can certainly be solved by computer if you can write the program to do it.

Starting at square a1 at the top left corner of the 7x7 grid shown, move one square at a time down or to the right until you reach square g7 at the bottom right-hand corner. Add up the numbers in each square you enter — including the first and last squares — to arrive at a total score for the path.

The object is to find the maximum and minimum scores possible, respectively. The solution is unique in each case.

Send the two answers on a postcard or the back of a sealed envelope to arrive not later than 30 November 1989. Send to: November Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG.

	1	2	3	4	5	6	7
a:	3	7	6	7	1	1	4
b:	8	2	9	5	9	4	5
c:	6	9	3	9	5	1	8
d:	5	4	6	5	6	8	8
e:	5	8	3	9	6	0	3
f:	5	6	5	2	8	4	2
g:	9	0	4	6	0	9	9