NUMBERS COUNT

PURSUITS

Some sequences generated using the Maximum Prime Factor function, presented by Mike Mudge.

This area of research has been suggested by a regular *PCW* reader, Bruce Halsey of Great Yarmouth, whose preliminary investigation is included below. There appears to be considerable scope for the generation of completely new integer sequences although the theoretical explanation for their behaviour may be difficult to discover.

It is known that a given positive integer, n, has a unique representation as a product of its prime factors; thus, $n = p_1^{m_1} p_2^{m_2} p_3^{m_3} \ldots\ldots p_r^{m_r}$ where m_r is the multiplicity of the prime factor p_r.

Note An efficient algorithm for obtaining this representation for a given n (not necessarily of multiple precision length) is desirable for this investigation.

The '**m**aximum prime **f**actor' function is defined as:
$mf(n) = \max(p_1, p_2, p_3, \ldots\ldots p_r)$, for example:
(**i**) $306761364 = 2^2 \times 3^3 \times 7^5 \times 13^2$ thus $mf(306761364) = 13$,
(**ii**) $1983009962 = 2 \times 101 \times 401 \times 24481$ thus $mf(1983009962) = 24481$.

The Halsey Sequence having prime seed p_0

This is defined for a given prime seed, p_0, by the recurrence relation A).....$p_{k+1} = mf(p_k^2+1)$; $k = 0,1,2,3\ldots\ldots$

Thus if $p_0 = 2$, $p_1 = mf(2^2+1) = mf(5) = 5$, $p_2 = mf(5^2+1) = mf(26) = 13$, $p_3 = mf(13^2+1) = mf(170) = 17$, the sequence continues as follows:
29, 421, 401, 53, 281, 3037, 70949, 1713329,.....

Bruce Halsey has used an Atari ST with Fast Basic and its assembler to investigate all p_0 less than 65535. He has found only *one* example of cyclical behaviour, *viz* $p_0 = 89$, $p_1 = 233$, $p_2 = 89$, etc, and thus poses the following questions:
1) Are there any other cycles?
2) Do all prime p_0 lead ultimately to the same sequence?
3) Are any Halsey Sequences unbounded? . . . The above example of p_0 equals 2 might suggest that they are; however, intuitively larger terms seem more likely to have more (smaller) prime factors.

Generalised Halsey Sequences may be defined in an obvious way, each with prime seed p_0.
B).....$p_{k+1} = mf(p_k^2 - 1)$,
C).....$p_{k+1} = mf(p_k^3 \pm 1)$ etc

D).....$p_{k+1} = mf(2^{p_k} \pm 1)$ etc

E).....$p_{k+1} = mf(3^{p_k} \pm 1)$ etc,
where $k = 0,1,2,3, \ldots$

For each of these sequences, questions regarding cyclical and unbounded behaviour can be asked.

Attempts to generate Halsey and Generalised Halsey Sequences and hence, or otherwise, to describe and explain their behaviour may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 March 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work on the problem, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, July 1989, Factorial Functions

A detailed study of some of the problems by Reg Bond of 75 Laburnum Crescent, Allestree, Derby DE3 2GS, earns this month's prize. Results obtained include:

Problem 2 Using a combination of modular arithmetic and computing in Fortran Integer*4 arithmetic, there are no solutions for n between 7 and 20,000. Those solutions for n = 3, 4, 5 & 6 are well known.

Problem 5 Reg first considered the modified problem: given m, how small can we make d? He established the theoretical answer that by increasing n and hence r, d can be made as small as we please. A 10-digit display pocket calculator yielded results including:

m = 5: n	**8**	**10**	**12**	**14**
d	**1.99**	**−0.776**	**0.212**	**0.025**

Other submissions deserving mention are: John McCarthy who used QL Superbasic on problem 1 and simultaneously his 'very under-used' Psion Organiser in 12s.f. arithmetic on problem 2. Computing speed proved a severe handicap in this investigation. Gareth Suggett who examined problem 1 for m,n less than 12; problem 2 for n less than 32; referred problem 3 to *PCW* September 1984; problem 4 results included the quadruple products for 8! & 9! and, finally, in problem 5 as Reg Bond, Gareth found the error in Croft's result for n = 20 where d = +0.126.

Review, Lucas Sequences, February 1989, re-opened August 1989

A difficult investigation to judge. The winner, by a very short head, is JR Wordsworth of 30 Branson Crescent, Melton Mowbray, Leicestershire LE13 1ER. The results were obtained on an Atari 520ST (½ megabyte + one single-sided DD) together with Devpack-Atari assembler package (Hisoft), Personal Pascal-compiler by OSS and certain public domain software including Forth 83 and C.

The investigation established Fibonacci primes up to U(1410) in about three days; Williams HC is still comfortably in the lead with U(2971) although this was hinted at by the other 'front-runner', Frank Webster.

The Lucas prime search showed that V(803) has prime factors 199, 151549 and 11899937029 and is *not* prime as stated by Paulo Ribenboim, *The Book of Prime Number Records*, page 287. This should undoubtedly read V(863) with V(1097), a possible candidate to join the other large Lucas primes V(503), V(613) and V(617).

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Jan 90

A Happy and Prosperous New Year to all. Maybe 1990 will be the year in which *you* win the Prize Puzzle!

Winner, October Prize Puzzle

Many of the 200-odd entries received complained that the problem was too easy. As some of you pointed out, the problem gave 7 equations to find only 5 unknowns. Still, it makes a change to have an easy one.

The missing symbol was the ampersand (&) which had a value of 10. The winning card, drawn from the heap, came from James Grinter of Grantham, Lincs. Congratulations, James, your prize is on its way.

This month's quickie

What is the closest relation that your mother's brother's brother-in-law could be to you?

Prize Puzzle

To start the year off, here's a problem in logic which should get the brains and perhaps the micros whirring.

The Foolem Insurance Company held its annual conference on the first day of March, this year. Each staff member had to bring one, and only one, partner.

When the conference opened at 2pm the only Board member to have shown up was the Finance Director with his wife. At that point, those present (there were less than 100) were divided into groups with five people in each.

By 3pm two more people had arrived — the Production Director and his wife. All those present were now subdivided into working parties with exactly four people in each.

By 4pm another two people had come — the Sales Director and his wife. People were now split into groups with three people in each.

By 5pm a further two had arrived — the Managing Director and his wife.

Because of a business meeting, one of the directors had intended that his wife should go to the conference without him and then he would join her an hour later. Fortunately, his business meeting ended early and the arrangement became unnecessary. However, had he done so, it would have been impossible for the people present to subdivide into smaller-sized groups. Which Director was it?

Answers on a postcard or the back of a sealed envelope (no letters, please) and send it to: January Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 January 1990. ■

$n \equiv 0 \pmod 5$
$n+2 \equiv 0 \pmod 4$
$(n+4) \equiv 0 \pmod 3$
$\therefore n = 50$
and $n-1, n+1, n+3, n+5$ is prime
49 ✗ 51 ✗ 53 ✓ 55 ✗
⇒ Sales Director