NUMBERS COUNT

A route optimisation problem with a difference, presented by Mike Mudge.

This area of research has been suggested by Jim Gibbons of Thornton Heath, Surrey and is absolutely unique in the seven-year history of 'Numbers Count'. There is not a single algebraic or mathematical symbol in the description of the problem, nor is there believed to be any published material (either empirical or theoretical) relating to its solution. Nonetheless, with a slight stretch of the imagination it can be regarded as a practical problem from the real world of engineering design.

Route optimisation usually involves finding, in some sense, the shortest path from one point to another using an existing network of roads, pipes, cables, and so on. An alternative and well explored problem area is that of connecting in some 'best possible' way a given finite set of stations, vertices, nodes, and so on.

Jim proposes the following: A new town is being designed on a square of unit side. The planners wish to first build a system of roads (negligible in area compared to the available land) all connected but not necessarily to the perimeter. It is required that no point of the interior of the square shall be more than half a unit away from a road and that further, for reasons of economy, the shortest total length of road is to be built. What configuration of road should be built?

Some possible ideas:

(a) Replace the square by a finite mesh of points.

(b) Restrict the roads to straight sections.

(c) Define a system of roads empirically using common sense and symmetry.

(d) Test the distance criterion and evaluate the total length with a computer algorithm.

(e) Iterate towards an optimum in some way?

Jim suggests the following possible generalisations:

(I) What happens if the minimum distance is reduced to one quarter of a unit, or indeed any other fraction? Is there a describable generalisation? Prove that any optimal solution must be symmetrical.

(II) Suppose the area of land available was circular and of unit radius (area), or indeed hexagonal as in many well known tiling problems. How would the solution be obtained?

Attempts to generate solutions to Jim Gibbons' problem may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141 to arrive by 1 April 1990. Any communications received will be judged using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work on the problem, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, August 1989: The Maltese Factor

This problem proved to be by far the most popular yet in both the multiplicity and variety of responses generated, so selecting the prizewinner was consequently a difficult task.

Many correspondents proved the conjecture of Albert Debono in Problem 1 and produced the appropriate computer program. A typical example was JC Hawdon of London, who continued in MacModula-2 on a Macintosh Plus to find the smallest integer expressible in n ways up to n = 47, yielding $45045 = 3^2 \times 5 \times 7 \times 11 \times 13$ in something over two hours.

In complete contrast Dr P McMullen of University College, London required less than two sides of typed A4 to provide complete theoretical analysis of both Problems 1 and 2. He believes that 'Problem 3 may be even deeper than the as yet unsolved Goldbach conjecture, that every even number greater than 2 is the sum of two prime numbers.' Interested readers are referred to 'Numbers Count', *PCW* September 1989.

Colin Singleton used Acorn Basic V on an Acorn Archimedes to display the smallest number which can be expressed as the sum of a series of consecutive positive integers in a specified number of ways, with its factors, up to 575 ways. However, he was able to avoid the problem using consecutive primes!

All entries received have been examined and a sample number forwarded to Albert Debono in Malta. Letters sent to him care of 'MRM at *PCW* will be forwarded immediately.

The final word on The Maltese Factor this month must remain with the worthy prizewinner: Peter Cameron of 70 Godstow Road, Wolvercote, Oxford OX2 8NY. In what he modestly describes as 'a sort of submission to your problem' Peter uses Turbo Pascal V4.0 run on a Zenith Z170 portable.

The extensive computation culminated with the conjecture that the average number of representations as a sum of consecutive primes tends to a constant, the icing on the cake being a theorem, with proof:

The average number of representations of the first x natural numbers as sums of consecutive primes tends to the limit $\log_e 2$ as x tends to infinity.

Well done, Peter!

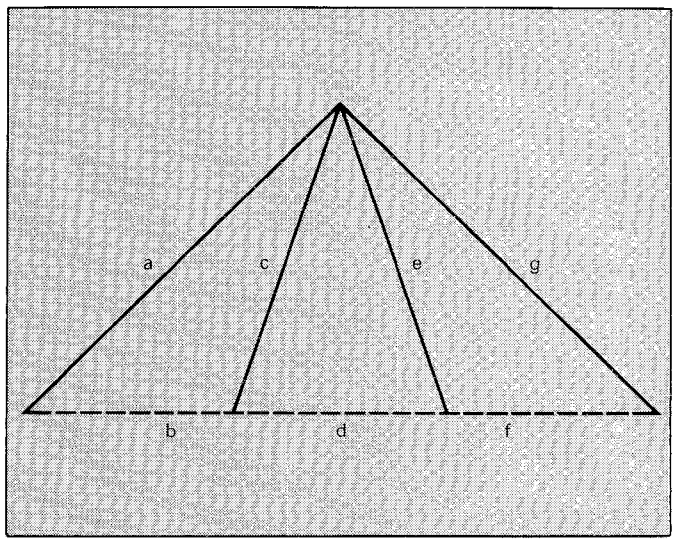
Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This Month's Quickie Feb 90

If you could fold a sheet of rice paper one thousandth of an inch thick exactly 50 times, how thick would the resulting wad be? Try doing it on your calculator — you'll be surprised at the answer.



Prize Puzzle

A puzzle to start the year off. It was sent to us by Mr Anthony Isaacs of London, and we like it very much. Thanks, Mr Isaacs — we just hope your answer is correct!

In the diagram above (which is not to scale) sides b,d, and f form a straight line, and each side has a length which is a prime number less than 500 — with no two sides being the same length. What is the sum of all the seven sides, given that it is greater than 1000?

Answers on postcards of backs or sealed envelopes — no letters please. Send to: February Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 February 1990.

Winner of November 1989 Prize Puzzle

Exactly 200 entries were received, but not all had the correct solutions which were:

Maximum Score: 89

Minimum Score: 52

The problem lent itself very conveniently to computer solution because of the relatively few permutations possible.

The winning card, drawn at random from the pile, came from T Patterson of Leatherhead, Surrey. Congratulations, your prize is on its way. To the remainder of the entrants, keep trying. It could be your turn next. ■