NUMBERS COUNT

Keeping up with the neighbours

Mike Mudge poses problems generated by the quaintly named Mathematics of Neighbourliness.

This area of investigation was first suggested by Martin Gardner in *Scientific American* some 15 years ago. However, in its present form I am indebted to Michael Meierruth of Milan who describes it as 'representing some good, clean programming fun involving some cute data structures and search algorithms.'

The concept is simple: given an N x N square grid it is required to place 2N pieces onto it in such a way that no three of them lie on a straight line of any orientation — that is, upwards, sideways or diagonally.

The first stage in the investigation is to find *a solution* for N as large as possible. It is at this stage that Michael Meierruth issues a challenge to all 'Numbers Count' readers: he, together with two other persons, responded to Martin Gardner by finding a solution for N=16, but he has recently dug out his code again. Following a four-day run on a Compaq Deskpro 386/20e a solution for N = 17 was forthcoming; the challenge is simply find a solution for N greater than 17.

An obvious extension of this work is to analyse the sequence of integers, counting the number of distinct solutions (not counting rotations and mirror images) as a function of N. Is this a finite sequence? That is, is there a maximum N beyond which no solutions exist?

Returning briefly to the title, the only obvious practical application is that if houses are arranged in this way on a square grid, then from each house some piece of every other house on the estate can be seen. This surely is neighbourliness!

A conceptually difficult extension which has occurred to the writer is that of pieces located within an M-dimensional hyper-cube. However, it is clear that a detailed investigation of the proposed 2-dimensional problem is essential before this generalisation is even considered.

Attempts to generate solutions for any values of N (and M) may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 May 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, Goldbach's Conjecture
Gareth Suggett defines G(n), n even, as the number of Goldbach decompositions of n and computes this in two distinct ways.

(1) Useful only for small n; given a table of primes add these in pairs and count the number of 'hits' on each value of n. Print out the total number of hits once all the pairs of primes being considered have sums greater than n.

(2) Partition n = A + B and then check A and B for primality. This was run up to G(1000) = 32.

A combination of these algorithms took Gareth to $G(10^6) = 5402$ at which point he found G(n) approaching 0.6 of the Hardy and Littlewood asymptotic formula for G(n) (see either Guy, *Unsolved Problems in Number Theory*, p58, or Ribenboim, *The Book of Prime Number Records*, p321). Can any reader either confirm or fault this result? It appears to be particularly interesting.

Gareth then modified his programs to consider only primes belonging to prime pairs and found the interesting fact that those numbers with no representations using this restricted set of primes appear to cluster in groups thus: 0,2,4;94,96,98;400,402,404; 514,516,518;784,786,788; . . .

However, using the usual philosophy of 'suitable subjective criteria', the prizewinner this month is WJ (Bill) Smith of Plymyard, The Barrow, Boddington, Cheltenham GL51 0TL. Bill first of all wishes to draw attention to the **non-profit making** operation known as I-APL Ltd of 2 Blenheim Road, St Albans, Herts AL1 4NR, whose stated aim is to spread awareness of APL as a problem solving tool. I-APL appears to be aimed at the educational market and is ported across Spectrums, BBCs and so on as well as PCs; it has been designed to exist within a 64k workspace and is therefore written for compactness rather than speed of operation.

Bill used I-APL on an Opus PC III with its 8088 running at 10MHz and went some way down the line with each problem posed, being constrained by I-APL's limited workspace on each investigation.

The language is built around array processing of 32k so does not provide a lot of space once the arrays get big. How might this be overcome?

Any replies direct to Bill please (or to the 'Letters' column of *PCW*) who will willingly supply copies of his programs to any interested readers.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Winner, December 1989
The Xmas number crossword proved to be quite popular. Well over 200 entries were received, mostly correct. But we could only have one winner, and that lucky person, drawn from the pile, was Mr IJ Inglis of High Wycombe, Bucks. Our congratulations to you, Mr Inglis, your prize is on its way. To everyone else, keep trying — maybe this month is your turn to win. The solution is given below.

Quickie
Our thanks to Miss Nicola Sparnon of Casterton School, Cumbria, for this month's puzzle.

Can you, by a single stroke of the pen (or pencil) make the following equation correct?
5 + 5 + 5 = 550

Prize Puzzle
This should burn up the micros:
16 contains 2 digits and it is also a perfect square.
125 contains 3 digits and it is also a perfect cube.
4096 contains 4 digits and it is also an exact 4th power.
59049 contains 5 digits and it is also an exact 5th power.
What is the largest number which has this property?

Answers on postcards or backs of sealed envelopes (no letters please) to: March Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 March 1990.

(1) 2	■	■	(2) 2	(3) 6	0	(4) 1	■	(5) 5	(6) 1	■	(7) 8
(8) 6	(9) 7	(10) 6	■	2	■	0	■	(11) 3	6	■	0
■	(12) 8	0	(13) 1	■	(14) 2	4	0	3	■	(15) 2	1
(16) 1	■	(17) 8	0	1	■	0	■	(18) 6	9	3	0
(19) 3	0	4	2	■	■	(20) 4	(21) 4	1	■	1	■
2	■	■	■	(22) 1	0	■	0	■	(23) 1	0	(24) 5
(25) 6	(26) 9	3	■	6	■	(27) 2	1	■	■	■	6
■	3	■	(28) 6	3	(29) 2	■	■	(30) 5	(31) 7	0	0
(32) 3	3	6	7	■	6	■	(33) 6	2	9	■	4
(34) 7	6	■	(35) 6	3	5	6	■	(36) 9	3	(37) 2	■
2	■	(38) 1	2	■	6	■	(39) 6	■	(40) 2	3	(41) 1
1	■	(42) 9	6	■	(43) 9	2	1	6	■	■	5