

Mike Mudge investigates Harshad Numbers and non-zero Harshad Numbers.

This topic first appeared in 'Numbers Count' in August 1983 on which occasion the associated prize award was withheld. Come along, PCW readers: there is no essential mathematical background other than basic arithmetic, but there is considerable scope for extending the current knowledge of this subject area.

A **Harshad Number** is a positive integer which is a multiple of the sum of its digits. For example, 4975 is a H-number because $4975 = (4 + 9 + 7 + 5) \times 199$, similarly 88998 is a H-number because $88998 = (8 + 8 + 9 + 9 + 8) \times 2119$.

A **Non-Zero Harshad Number** is a H-number, none of whose digits is zero. Thus both of the H-numbers shown above are NZH-numbers whereas 40975, although a H-number, is not an NZH-number on account of the zero digit which it contains.

The properties required for H-numbers and NZH-numbers are clearly **base-dependent** which makes them particularly interesting to the empirical number theorist using a personal computer. For example, 190 is a H-number since $190 = (1 + 9 + 0) \times 19$: it is not a NZH-number. However, if we move from base 10 to base 3 (in which, incidentally, the first generation of Russian computers worked, rather than base 2 or binary) we find $190_{10} = 21001_3$: that is, $190 = 2 \times 3^4 + 1 \times 3^3 + 1 \times 3^0$. The sum of the digits is now 4 and since 190 is not an integer multiple of 4 the property of being a H-number has disappeared. It returns again in base 4 where $190_{10} = 2332_4$, the digit sum again being 10. However, 2332_4 has no zero digits and is thus an NZH-number, unlike 190_{10} .

Project A Design and implement an algorithm for computing the smallest H-number, base 10, having digit sum d for $d = 10, 11, \dots$. Note that digit sum 1, ..., 9 is trivial yielding smallest H-numbers 1, ..., 9 respectively (see below).

- Questions of immediate interest include:
- (i) What are the smallest H-numbers with digit sums 40 and 41?
 - (ii) Is 291899999999 the smallest H-number with digit sum 101?
 - (iii) Is 8587 followed by 27 nines the smallest H-number with digit sum 271?

Project B Extend the algorithm of A above to find the smallest NZH-number when this is different from the smallest H-number. Careful!

Project C Extend the results of A and B above to any number base, b , to be input together with the required digit sum, d .

Conjecture to be considered: 'The number of NZH-numbers for a given d is finite' . . . If this is true, is it realistic to list all such numbers.?

Thought for the month

Several years ago Peter Stanbury, a numerologist from Tunbridge Wells, wrote to me with the following observation: 'If a, b, c , and d are all consecutive primes larger than 5 then the following formula produces a surprising number of primes: $ab - (c + d)$. For the first 18 values of 'a' the formula produces 15 primes, this ratio decreasing to 23 out of 31, 35 out of 67, and 48 out of 119. I have not checked it further, but you might wish to do so.'

Do readers agree with these numbers, wish to extend them, and is this a surprising number of primes?

Attempts at the above projects together with observations on the thought for the month may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel: (09746) 548 to arrive by 1 July 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by PCW to the 'best' contribution.

It would be appreciated if such submissions contained a brief description of the

hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in PCW.

Review, Greedy Sequences

A Parry of Porth used BBC Basic on an Archimedes A310 to find the first 258 terms of the Sidon Series less than 350000. Gareth Suggett did his usual comprehensive attack on all the problems but the total number of substantive responses was

disappointing.

The prizewinner, however, is a quartet of Portuguese students referred to as Luis Lampreia et al of Rua Jacinto Nunes 5-4 Esq. 1100 Lisboa, Portugal, using a Philips MSX-2 together with assembler on a Spectrum. Details of the group's work are available from the proposer, Peter Cameron, of 70 Godstow Road, Wolvercote OX2 8NY.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future articles.

LEISURE LINES

Brainleasers courtesy of JJ Clessa.

This Month's Quickie

No answers, no prizes. Can you find which of the following numbers is the odd man out? 968 583 375 286 781 605 209 946

Prize Puzzle

And now for a crossnumber puzzle with a difference. Our thanks to Peter Stanton of London for this novel idea.

Each answer to the 26 clues is a number in which exactly two identical digits are present — no more and no less. The clue itself shows the sum of the digits in the answer.

When you have completed the puzzle paste the completed grid to a postcard or the back of a sealed envelope — please don't send letters — and post it to: May Prize Puzzle, PCW Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 May 1990.

Winner, February 1990

Where were all those entrants who invariably cry 'Too easy!?' There were less than 40 entries for the February problem. One reader gave up . . . after 10 hours coding and 4 hrs 45 mins running on an Amstrad PC1640. . . . Another wanted to see a program for solving it since he estimated that it would need six months on his micro to do it. I'm sorry Mr N, but everyone obeyed the rules this time and no letters were sent (usually we get at least six or seven letters which invariably contain program listings), but since the puzzle was submitted by Mr Anthony Isaacs, perhaps he might wish to offer one.

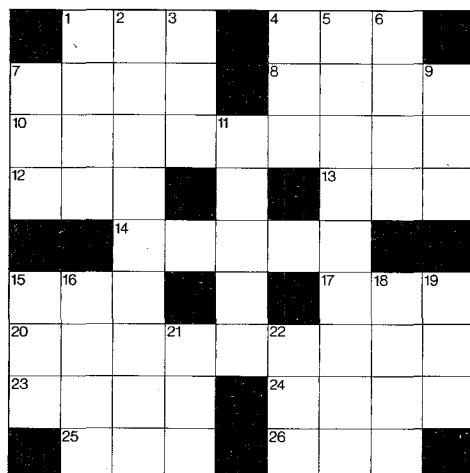
The answer to the problem, found after much number crunching, was 2101 with sides of 449, 151, 457, 17, 461, 79 and 487. The winning card came from Scotland — from Mr P Bradbeer of Inverkeithing, Fife. Congratulations, Mr Bradbeer, your prize will be sent forthwith.

Clues Across

- 1a 12
- 4a 17
- 7a 33
- 8a 7
- 10a 40
- 12a 8
- 13a 10
- 14a 35
- 15a 15
- 17a 21
- 20a 51
- 23a 12
- 24a 13
- 25a 21
- 26a 13

Clues Down

- 1d 16
- 2d 53
- 3d 12
- 4d 20
- 5d 37
- 6d 13
- 7d 9
- 9d 19
- 11d 21
- 15d 26
- 16d 22
- 18d 15
- 19d 18
- 21d 5
- 22d 6



Test Data

Digit sum	10	14	18	22	26	42
Smallest H-number	190	266	198	2398	1898	88998