Mike Mudge poses some problems in amicable and quasi-amicable numbers.

Given any positive integer, n, we define s(n) to be the sum of all of the proper* divisors of n (*that is, excluding n itself), hence s(6) = 1+2+3 = 6 while s(28) = 1+2+4+7+14 = 28.

Readers may recall that n is defined to be a 'perfect' number if and only if (iff) s(n) = n, ref *PCW* October 1983. Thus from the above examples both 6 and 28 are perfect numbers.

Extensions of this definition include the following:

1) n is **quasi-perfect** or **almost perfect** according to s(n) = n+1 or n−1 respectively.

2) n is **2−hyper-perfect** iff n = 2 × s(n) −1.
n is **3−hyper-perfect** iff n = 3 × s(n) −2. ....
n is **r−hyper-perfect** iff n = r × s(n) −(r−l).

3) n is **multiply perfect** with **index of perfection** k iff s(n) + n = k × n. For example, 120 is multiply perfect with index of perfection 3 while 30240 is multiply perfect with index of perfection 4.

4) $n_1, n_2, n_3, ...n_t$ are **sociable** numbers with **index of sociability** t, iff $s(n_1)=n_2$, $s(n_2)=n_3, ...s(n_t)=n_1$. For example, 14316 is a sociable number with index of sociability 28.

(*Note.* Perfect numbers are simply sociable numbers with index of sociability 1.)

The central topic for investigation this month focuses on sociable numbers with index of sociability 2.

Definition The pair of positive integers (m,n) is defined to be an **amicable** pair iff s(m) = n and s(n) = m; clearly equivalent to m and n being sociable numbers with index of sociability 2.

The first, that is smallest, amicable pair (220,284) has been known since ancient times. Dickson (1952) proved that there exist only five amicable pairs in which the smaller number is less than 6233, but by 1973, 1107 amicable pairs were known, the largest being a pair of 25-digit numbers.

Subsequently, four further amicable pairs (at least) have been discovered with 32, 40, 81 and 152-digit numbers. The existence of relatively prime amicable pairs, that is where the two integers have no common divisor other than unity, has been proved theoretically — Kanold (1953), Hagis (1969-'70). However, none have been found in a search up to 10^{60}.

Definition The pair of positive integers (m,n) is defined to be a **quasi-amicable** pair iff s(m) = n+1 and s(n) = m+1. Forty-six quasi-amicable pairs are known, with the smallest member less than 10^7. The sequence begins with (48,75);(140,195) and 'ends' with (8829792,18845855); (9247095,10106504).

Project A Design and implement a computer program to obtain amicable pairs, hence determine as many as possible of the 1107 such pairs referred to above.

Project B Design and implement a computer program to obtain quasi-amicable pairs and attempt to discover the 46 such pairs whose smallest member is less than 10^7. Extend this result if possible.

Project C Design and implement a computer program to tackle the more general problem of sociable numbers with general index, t.

Thought for the month: Things you may not have known about powers of 2. 2^{10} = 1024 is the smallest power of 2 containing a zero. 2^{53}, which is approximately 9.007199255 × 10^{15}, is the smallest power of 2 containing two *consecutive* zeroes. What is the smallest power of 2 containing *nine* consecutive zeroes? The result is known thanks to Mr C Tooth, *PCW* February 1986. What are the corresponding results for consecutive ones, twos,... and powers of 3,4,5....?

Attempts at some, or all, of the above projects together with reactions to the thought for the month, may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (09746) 548, to arrive by 1 September 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

Review January 1990

Thanks are due to Bruce Halsey of Great Yarmouth for a most interesting area of investigation. Several contributors dealt swiftly with $p_{k+1} = mf(p_k^2 - 1)$ which ultimately cycles at (2,3).

The only complete treatment of $p_{k+1} = mf(p_k^2 + 1)$, the 'fundamental' Halsey sequence, was due to Reg Bond of Derby who established that cycling can *only* occur around two distinct primes and that these are $p_0 = u_r$ and $p_1 = u_{r+2}$ where u_r and u_{r+2} are odd Fibonacci Numbers, r greater than or equal to 11 if (3, (r+4)/3) = 1 and r greater than or equal to 14 if (3, (r+4)/3) greater than 1..

The solution for r less than or equal to 1000 are (i) u_{11} = 89, u_{13} = 233 as quoted by Halsey, (ii) u_{431}, u_{433} with 90 and 91 digits respectively, and (iii) u_{569}, u_{571} with 119 digits each.

Gareth Suggett draws readers' attention to $p_{k+1} = mf(2xp_k + 1)$ where the cycle (5,11,23,47,19,13,3,7) 'appears to attract everything'.

However, the worthy prizewinner is Frank Webster of 125 Coniston Grove, Middlesbrough, Cleveland TS5 7DF, with extensive studies of all problems A)...E) using an Acorn Electron programmed in BBC Basic with some routines in assembly language. Space does not permit a detailed discussion of the results, but they may be obtained from Frank or myself, in summary, on receipt of a suitable sae.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This Month's Quickie

No answers, no prizes, but slightly more difficult than usual. On the island of Monga-Monga there is no such thing as money so the natives barter with fruit. The going price for three goats and a mule is two cows, and the going price for three cows, two mules, and a goat is 25 sheep. How many sheep will you get for each of the other three animals?

Prize Puzzle

And now for something slightly more difficult that should stretch the micros — or at least, the grey matter.

In a boys' school in Glasgow one of the classes contained exactly 30 boys, 15 of which were Celtic fans, the other 15 supported Rangers. As a punishment for a misdemeanour, for which the culprit could not be found, the teacher decided to detain exactly one half of the class over a Saturday afternoon period so that they would miss the 'derby' match. In order to be fair, he arranged the entire class in a circle and proceeded to count clockwise, with every 13th boy dropping out for the Saturday detention, until 15 boys had been removed. However, a clever Celtic supporter told his friends where to stand so that only the Rangers fans would be punished.

What are the positions (1 to 30) where the Celtic fans should stand at the start of the count so that none of them will miss the game?

Answers on postcards or backs of sealed envelopes — no letters please. Send to: July Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 July 1990.

Winner, April 1990

Due to an unfortunate glitch in the system, no puzzle appeared in the April issue. Due to the same glitch we didn't give the winner of the January puzzle, so we'll do that now. (Phew, and we thought our puzzles were complicated!)

Winner, January 1990

There was a mediocre response to our January problem and yet it was quite easy — perhaps that was the trouble. Less than 80 entries were received, about 10 of which were incorrect. The answer to the problem was the Sales Director, and the winning card drawn from the batch came from Nairobi, Kenya — from Mr (?) Agola Gregory. Congratulations, Mr Gregory, your prize will be sent forthwith. To the also-rans — keep trying, maybe this month is your turn.