Part two of quadratic partitions and cubic residues explored by Mike Mudge.

This month readers are invited to continue the exploration of this fascinating research area suggested by N V Meeres of Esher and introduced as Part I, *PCW* June 1990. The pure mathematical concepts required are carefully defined, with simple numerical examples, in Appendix A, reproduced here from Part I for completeness.

Throughout this work, p denotes a prime number congruent to 1 modulo 6, and hence having associated unique quadratic partitions (QPs) of the form $\mathbf{p = 3xA^2 + B^2}$ **and** $\mathbf{4p = 27xC^2 + D^2}$ (Part I, Project A).

Now consider the composite qr = 6 and ask when this is a cubic residue (CR) of p. The QP $27xC^2 + D^2$ becomes useful in cases where neither 2 nor 3 is a factor of CxD. It can be shown that under these conditions either C + D or C − D must be congruent to 0 modulo 6. If the quotient on division of C ± D is even, then 6 is found to be a CR of p; otherwise 12 is a CR and 6 is not.

Other values of 2q, such as 22 and 34, behave in the same way, provided that, like 6, they are congruent to ±1 modulo 7 and that C + D or C − D are congruent to 0 modulo 4q. Here the primes to be considered will include some where C is divisible by 3, but 2 and q must not be factors of either C or D.

Project A Investigate the sequence of primes **7, 13, 97, 139, 151, 313, 673, 751, 769, 937, 1063, 1117, 1321,** . . . using the above results to determine possible CRs. Hint: Results should begin **(6),(12),(12),(6,44)** (44 also 3 & 19 by part I), . . .

Now if 2q ≡ ±3 mod 7 it is a CR when C ± D ≡ 0 mod 2q but C ± D ≢ 0 mod 4q. Similarly, if C ± D ≡ mod 4q then 4q is a CR but not 2q.

Project B Investigate the sequence of primes **73, 97, 103, 193, 349, 373, 571, 661, 673, 757, 1123, 1429, 1543,** . . . using the above results to determine possible CRs. Hint: results should begin **(10),(20,12),(10),(20),(38,6)** . . .

Now if 2q ≡ ± mod 7 it cannot be a CR of this class of primes, for it is found that q itself is a CR whenever C ± D ≡ 0 mod q.

Project C Investigate the sequence of primes **163, 709, 877, 991, 1783, 1867,** . . . using the above results to determine possible CRs. Hint: results should begin **(13),(13),(29),(29),(43),(41),** . . . Recall that the object of this investigation is to attempt to generate a complete set of CRs for any given prime number congruent to 1 modulo 6. With this object in mind readers are encouraged to consider, in both theory and practice:

Project D For the case 2q = 14 the key to the analysis is the QP $4p = 3XE^2 + F^2$ where E = A ± B whichever is not congruent to zero modulo 7. How does this key provide information on CRs?

Attempts at some, or all, of the above projects may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (09746) 548 to arrive by 1 October 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date.

APPENDIX A Some pure mathematical concepts and definitions.

(i) Modulus & Congruence Two integers m & n are said to be congruent modulo a third integer r if and only if (iff) they differ by a multiple of r: that is, m & n leave the same remainder on division by r. We write **m ≡ n(r)**, or **m ≡ n mod.r**, for example **17≡65(4)** because **7-65 = -48 = −12x4.**

(ii) Quadratic Partition (QP) A QP of a given integer, K, is simply an expression of K as the sum of multiples of two squares, for example $\mathbf{68 = 3x4^2 + 5x2^2}$ is a QP of 68.

(iii) Cubic Residue (CR) If p greater than 2 does not divide, a, and there exists an integer n such that $\mathbf{a \equiv n^3(p)}$ then a is a CR of p. For example, if **p = 19** then $\mathbf{11 \equiv 5^3 (19)}$ and 11 is a CR of p. Notice that **11 − 125 = −114 = −6x19.**

(iv) Order The order of an integer, a, modulo p is defined as the smallest power of a which is congruent to 1 modulo p.

> Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles.

LEISURE LINES

JJ Clessa's brainteasers.

Prize Puzzle Aug 90

This is a micro-whirling puzzle which I used once before, many years ago.

Three friends, Alan, Bert and Colin, possess a Mercedes, a Metro, and a Moped respectively. The three are discussing their vehicles' mileages when Alan reports that the 6-figure speedometer on his car is showing a palindromic value of 006600 miles. 'That's interesting,' says Bert, 'mine is also palindromic — 18981 miles, although my speedometer only has 5 figures'. 'Well, I never' says Colin, 'my 4-figure Moped speedo is showing 5335 miles, so we're all palindromic at the same time — I wonder if we're ever likely to get such a coincidence again'.

Well, of course, we could never answer that since each vehicle does a different weekly mileage from the others. But we do have four different questions for you. Supposing that all three speedometers were fitted to Alan's Mercedes and were equally accurate, what is the least number of miles that would elapse before:

1 Alan and Bert's mileages were mutually palindromic again.

2 Bert and Colin's mileages were mutually palindromic again.

3 Alan and Colin's mileages were mutually palindromic again.

4 All three mileages were mutually palindromic again.

Answers on postcards or backs of sealed envelopes — no letters, please — to: August Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 August 1990.

■	(1) 5	(2) 5	(3) 2	■	(4) 9	(5) 4	(6) 4	■
(7) 7	9	9	8	■	(8) 2	1	1	(9) 3
(10) 1	1	4	2	(11) 5	9	3	7	8
(12) 1	1	6	■	1	■	(13) 1	1	8
■	■	(14) 7	9	8	9	2	■	■
(15) 9	(16) 3	3	■	1	■	(17) 8	(18) 5	(19) 8
(20) 9	9	8	(21) 1	6	(22) 4	7	2	5
(23) 8	1	2	1	■	(24) 1	5	2	5
■	(25) 9	9	3	■	(26) 1	6	6	■

Winner, May 1990

A good response to the unusual number crossword in the May puzzle — and most people seemed to enjoy it. Almost all of the 80 entrants submitted the correct solution, so it was up to our random selection process (find the postcard with the fivers attached) to give us our winner, who this month is Mr FC Jessop of Bridport in Dorset. Congratulations Mr Jessop, your prize will be sent to you shortly. Our usual condolences to the also-rans with our usual plea — don't give up.

The correct solution is shown.