NUMBERS COUNT

Got a problem with Babylonian fractions? Mike Mudge has five — see if you can solve them.

This month's area of investigation has been suggested by Dr Nigel Backhouse of Helsby, Warrington, Cheshire, who relates the problem to work done by the Babylonians some three and a half to four thousand years ago. The mathematical prerequisites are nothing other than the algebraic rules for manipulating fractions, but there appears to be considerable scope for ingenuity of programming. Nigel describes himself as 'by no means an expert in number theory, even less so with anything to do with computers! . . . but having a general interest in algorithms.'

The Babylonian notation for a fraction represented only those fractions of the form 1/N. Thus in order to use a more general fraction it had to be split up into a sum of such unit fractions. For example: $5/6 = 1/2 + 1/3$; $7/20 = 1/3 + 1/60$; $9/20 = 1/4 = 1/5$.

Problem 1 Given a proper fraction P/Q (that is, where P is less than Q and the pair P,Q have no common integer divisor) express it as a sum of unit fractions. *Note.* Ignore the trivial solution where 1/Q is added to itself P-times.

Possible algorithm for this problem

F (0) = P/Q,
F (K+1) = F (K) – [the largest unit fraction less than or equal to F (K).]
The sequence F (0), F (1), F (2)...tends to zero and so (1)...P/Q = [F (0) – F (1)]+F (1) – F (2)] +...gives a possible solution.

For example: $5/6 = 1/2 + 1/3$; $7/20 = 1/3 + 1/60$; $9/20 = 1/3 + 1.9 + 1/180$.

It is to be observed that the representation (1) above must be finite (that is, terminate) because the numerators of the fractions decrease strictly monotonically, and eventually unity must be reached. Thus at most P unit fractions are needed, this is sometimes the case, for example $2/3 = 1/2 + 1.6$.

Problem 2 Extension of Problem 1 above and its suggested algorithmic solution to any irrational number, lying between 0 and 1, to obtain a non-terminating representation as a sum of unit fractions. Can any use be suggested for this representation?

Problem 3 Given that for P/Q the expansion in unit fractions is not unique, *viz* $9/20 = 1.3 + 1/10 + 1/60 = 1/4 + 1/5$, and that further the above algorithm does not necessarily yield the shortest expansion: is there an algorithm which always yields the expansion with the smallest number of unit fractions?

Problem 4 How best does one obtain lists of expansions of P/5, P/6, P/7 and so on. Is there an underlying pattern? Does anything special happen when Q, the denominator throughout this work, is a prime number?

Possible Clue? If one has expansions of P/Q and R/S then one can immediately write down (some?) expansions of (PR)/(QS). Nigel suggests here that 'A large bank of examples might help', for example $(5/6)(7/20) = 7/24 = (1/2 + 1/3)(1/3 + 1/60) = 1/6 + 1/120 + 1/9 + 1/180$.

Problem 5 How does the theory of representation of irrational numbers, lying between 0 and 1, as non-terminating sums of unit fractions vary with number base used for the arithmetic? For example, how does the accuracy of a truncated representation depend upon the number base?

Attempts at some or all of the above problems may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel 0974-272548, to arrive by 1 April 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, August 1990

The response to this article, the second of three parts due to Norman Meeres of 89 Grove Way, Esher, Surrey KT10 8HF, included a very detailed study by Frank Webster of Middlesborough. He examined project A with $p < 10,000$ in 65 minutes, project B with $p \leq 9967$ in 67 minutes, project C with $p < 10,000$ in 42 minutes and project D with $p < 20,000$ in 120 minutes. After much soul searching the prizewinner this month is Norman Meeres for his totality of effort divided over the three parts of this study.

As promised in *PCW* December 1990 an overview of the relationships between quadratic partitions and cubic residues can be expected in June 1991 when all relevant submissions have been analysed and a closing comment from Norman obtained.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for further Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

Here's a February quickie for you — no prizes, no answers. When Albert and Ken ran a 100 yard race, Albert won by 4 yards. Next day they raced again, but this time Ken got 4 yards start. Assuming they both ran at the same speed as before, who would be the winner? (The answer is *not* a dead heat.)

Prize Puzzle

And now for a real number cruncher, although it shouldn't prove too difficult. There are three parts:

A certain number contains n digits, and the sum of the digits raised to the power n equals the number itself. To illustrate:

The 3-digit number
$153 = 1^3 + 5^3 + 3^3$
The 4-digit number
$1634 = 1^4 + 6^4 + 3^3 + 4^4$

Can you find a 5-digit, a 6-digit, and a 7-digit number with the above property *and* which between them use every digit from 0-9 except the digit 7?

Send your three answers on postcards or backs of sealed envelopes (no letters please) to February Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 February 1991.

Winner, November

The problem of the number of triangles created by joining every vertex of a regular octagon proved a much tougher puzzle than usual. Only 46 entries were received and half of those were incorrect.

The winning entry came from Chester, from Mr GE Reynolds. Well done, Mr Reynolds — your prize is on its way. The correct number of triangles was 632.

Don't give up — it could be your turn next. ■