Mike Mudge presents you with an opportunity to use your Random Number Generator as he poses a problem with dice.

This month's problem is based upon a suggestion by Neil Duncan of Ealing, London W5 2AD. It is the sort of problem that may be readily soluble by the technique of manipulating 'probability generating functions'. However, here it will be used to show how a simple probability situation can be easily generalised to generate inconveniently large numbers and disarmingly simple questions. It will also be used to illustrate the limitations of (micro)computers, namely their inability to think!

If player A throws a dice (assumed throughout to be an unbiased cubical dice numbered 1-6) and offers player B odds of 5-1 against getting the same score with one throw, this is clearly a fair bet. If nowever player B is permitted a second throw, and then to add the scores together if the first one is too low and then again if the total is still too low, and so on, so that B is trying to make a *total* score equal to the *single* score of A, would odds of 3-1 be fair? Or suppose that player A threw several dice simultaneously, with the *total* score the target for player B with no limit on the number of throws, would odds of 5-2 be fair?

Consider first the case where player A throws only one dice with scores of 1,2,3,4,5,6 all equally likely with probability 1/6. With a throw of 1 player B can win in one way only, *viz* by getting a 1 first throw, a 1/6 chance. However with a throw of 6 B can win in one way with one throw (6), in one way with six throws (1+1+1+1+1+1), in five ways with two throws (1+5,2+4,3+3,4+2,5+1) or with five permutations of 2,1,1,1,1 and in ten ways with either three or four throws. So, the overall chance of scoring six is given by 1/6 + 5/36 + 10/216 + 10/1296 + 5/7776 + 1/46656 which totals 16807/46656 or approximately 0.360232. This discussion covers the chances of B winning when A throws 1 or 6. A complete investigation averaging over all six results yields 70993/279936 or approximately 0.253604 so the suggested odds of 3-1 are essentially fair.

The same principles can be applied to player A throwing two or more dice but they soon become tedious. For example, with two dice scores from 2 to 12 are possible and the intermediate totals are not equally likely. Further, in response to a score of 12 there are 1936 ways for B to win, one of which (twelve ones) has a probability of only 1 in 2176782336. A complete analysis of this case has been carried out by Neil Duncan with a result of approximately 0.2835. Would the chance of B winning improve if A used more than two dice?

Let D denote the number of dice thrown by player A, recall that B continues to throw one dice until either winning (that is, equalling the total score of A) or losing (that is, exceeding the total score of A); the table below shows the probability, p, that B wins correct to nine decimal places.

The immediate question is: Does this probability tend to a limit as D increases? Almost self evident, is this limit equal to 2/7? If so, why, so if not, why not?

Problem 1 Construct a computer program to evaluate p as a function of D, wherever possible exact arithmetic should be used. The proposer observes that tabulation of the numbers of ways of scoring S with D dice soon reveals a pattern that can be produced by applying simple rules. 'If you teach the computer the rules, it will happily calculate your chance of B winning with *any* number of dice in use by A.'

Problem 2 Simulate this dice game using a random number generator and obtain empirical values of p as a function of D. How many trials are required to obtain a given accuracy in p? There is a suggestion that 100,000 trials give p correct to two decimal places!

Compare and contrast the two approaches to the limiting value of p.

Attempts at either or both of the above problems may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel 0974-272548, to arrive by 1 May 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

D	1	2	7	8	12
p	0.253604395	0.283539658	0.285714342	0.285714282	0.285714284

Mike Mudge welcomes correspondence on any subject within the area of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

LEISURE LINES

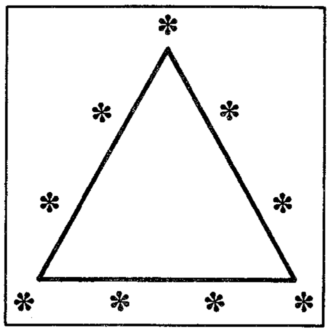
Brainteasers courtesy of JJ Clessa.

This Month's Quickie

A nifty quickie for March — no prizes, no answers. A hiker walks at 2mph uphill and 6mph downhill. He walks up a hill, turns straight round at the top, and walks back down. What is his average speed for the trip? (Don't say 4mph — it's the wrong answer.)

Prize Puzzle

And now for something different — a logic puzzle which you may be able to crack with or without the help of your micros. Draw two



equilateral triangles and place along the sides of each, nine digits in the positions shown above, so that:

1) No digit may appear twice on the same triangle.
2) The sum of the digits along any side of a triangle is 14.
3) All 10 digits (0-9) are used.

Which two digits appear only once?

Send your answers on postcards or backs of sealed envelopes — no letters please — to: March Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 March 1991. Good luck!

Winner, December 1990

I'm afraid we made a small error when we set this problem — we omitted to restrict the digit multiplier to values greater than unity. In consequence, any 6-digit palindrome with a multiplier of 1 would have been valid. But although many of you pointed this out to us, all the 75 entries received adhered to the spirit of the problem. The two required solutions were therefore:

$109989 \times 9 = 989901$
$219978 \times 4 = 879912$

and this month's winner was Mr A Richardson of Croxley Green in Hertfordshire. Congratulations, Mr R, your prize will be with you shortly if not sooner. Our usual condolences to the also-rans. Don't give up — it could be your turn next.