Mike Mudge poses some problems in repetends, presents a result due to Shrader-Frechette, and examines sum multiples.

A proper fraction is defined as the quotient of two positive integers, p/q, p less than q. The decimal expansion of such a fraction is either (a)finite (terminating) or (b)infinite (recurring). For example, (a) 3/5=0.6, (b) 3/7=0.428571428571 .... written 0.(428571). The ordered set 428571 is called a *repetend* of *length* 6.

Now, a periodic decimal of length 2n can be converted to a fraction by dividing the *repetend* by $10^{2n}-1$, for example 0.(428571)= 428571/999999.
Also, when the sum of the number, A, formed by the first n digits of the repetend and the number, B, formed by the second n digits is 10^n-1, the conversion is 1+A divided by 10^n+1, for example 0.(428571)=429/100.

Consider sums of partitions of the repetend of certain fractions when the partitions consist of r blocks each of n digits. For example, 1/13= 0.(076923), r=2, n=3, 076+923=999. r=3, n=2, 07+69+23=99. r=6, n=1, 0+7+6+9+2+3=27*. (*Now there is a total with greater than n digits so add the *excess* left-most string of digits on the right to the right-most block of n digits.) Thus, 2+7=9. When the result of this process is a repeated digit (*repdigit*) consisting of n 9's the fraction is defined to be *complementary* of order n,r.

M Shrader-Frechette (*Mathematical Magazine*, 1978) showed that all proper fractions are complementary for *any* n, provided that sufficient digits of the *extended repetend* are used. For example, 0.(076923), n=4, 0769+2307+6923 =9999. n=5, 07692+30769+23076+92307= 299997*. (By * above 99997+2=99999.)

Given a prime, q, the length of the repetend of 1/q is denoted by L(q) and the quotient (q−1)/L(q) or *residue index* by i(q).

Sum multiples
Example q=41, L(q)=5, i(q)=8. Consider the q−1=40 proper fractions with denominator 41. These can be partitioned so that they correspond to 8 disjoint loops of permutations of the repetend. One such loop is: 1/41=0.(02439), 10/41=0.(24390), 18/41=0.(43902), 16/41=0.(39024), 37/41=0.(90243).

Within such a loop the sum of the numerators is a multiple of the denominator, this is the *sum multiple*. In the above loop 1+10+18+16+37= 82=2×41, *sum multiple* 2. For example, if q=4649, L(q)=7, i(q)=664, there are found to be 262 pairs of loops with sum multiples of 3 & 4, 66 pairs with sum multiples of 2 & 5 and 4 pairs with sum multiples of 1 & 6.

Problem 1 Construct and implement an algorithm to verify the result of M Shrader-Frechette.

Problem 2 Construct a table of primes that have an even residue index and an odd repetend length; including details of the sum multiples of their loops.

Supposing that there are 'a' loops with sum multiples m_1 & m_2: see the test data in the box below.

Attempts at one or both of the above problems may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (Nebo) 09746-548, to arrive by 1 June 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, together with sufficient details of programs to enable the results to be reproduced. Run times, a summary of results obtained and suggestions for further work in this area should be enclosed, in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, October 1990
This subject area was suggested by Michael Meierruth of Milan. Michael subsequently pointed out (22/11/90) that he was 'a bit hasty about concluding that the difference between sequential copying and an ideally rearranged sequence would result at worst in an extra floppy. This is obviously not true. If we take a floppy capacity of 3600 and have files with sizes 1800, 1, 1800, 1, 1800, 1, 1800, 1, 1800, 1, 1800, 1, then this will obviously take 6 floppies whereas the best rearrangement would use only 4 floppies. This difference widens the longer the sequence.'

However, the prizewinner this month is John Tissandier of 17 The Avenue, Tiverton, Devon EX16 4HP. John's algorithm does a linear search of the files, copies the largest onto the first floppy and calculates the space left. Then a linear search of the remaining files finds the largest to fit the residual space, and so on until no files small enough are left; the algorithm then changes to the next floppy. A *very* simple but well performing algorithm implemented in Turbo Pascal Version 4 and incorporated in a larger program allowing the user to decide on a number of filesets each containing a certain number of files to be copied, together with a random data generation routine for test calculations.

q	L(q)	i(q)	a	m_1	m_2
31	15	2	1	6	9
239	7	34	4	2	5
			13	3	4

Reference: Samuel Yates, *Repunits and Repetends*, Star Publishing Co, 1982.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This Month's Quickie
What is weightless, can be clearly seen, and if put in a barrel, will make the barrel weigh less? Easy — or is it?

Prize Puzzle
This one shouldn't be too difficult. In a certain village, Mr Anson, Mr Benson, Mr Carson, and Mr Dobson are Accountant, Baker, Chemist, and Doctor, but not necessarily respectively. Each man's monthly income is an exact number of pounds (no pence).

1 The Chemist earns exactly twice as much as the Doctor, the Accountant earns exactly twice as much as the Chemist, and the Baker earns exactly twice as much as the Accountant.

2 Mr Benson earns more than Mr Anson but not twice as much.

3 Mr Dobson earns exactly £3766 more than Mr Carson.

What is each man's occupation, and how much does Mr Anson earn per annum?

Send your answers on postcards or backs of sealed envelopes — no letters please — to: April Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive no later than 30 April 1991. Good luck!

Winner, January 1991
We had about 70 entrants to our simulation problem. Only half of these actually used simulation, the other half had obviously heard of Georges Buffon who first presented this phenomenon. For phenomenon it is, since the result obtained approximates to 8/pi.

The winner (chosen by throwing a stick on a shove-halfpenny board) was Mr Jeremy Thorp of Bristol. Congratulations, Mr Thorp, your prize is on its way. To the also-rans, don't give up — it could be your turn next.