Mike Mudge the magician examines magic squares.

This topic was first introduced to *PCW* readers in October 1987. The return visit is inspired by *Invitation To Number Theory with Pascal*, Donald D Spencer, Camelot Publishing, 1989. This excellent book includes a 25 page treatment of magic and related squares: 'Not only are magic squares fun to construct, they also provide excellent programming exercises for the microcomputer user.'

A *simple magic square*, of order n*, is defined to be a square array containing the integers from 1 to n^2 and such that the sum of each row, column and principal diagonal is a constant. (This *magic constant* clearly has the value $C_1 = n(n^2 + 1)/2$.) The earliest known magic square, called the lo-shu, is attributed to the Chinese Emperor Yu, who, in about 2200 BC, is said to have found a symbolic representation of

```
8  1  6
3  5  7
4  9  2
```

where n=3 & C_1=15, on the back of a tortoise.

*Magic squares are said to be of odd order if n = 2m + 1, of singly even order if n = 2(2m + 1) and of doubly even order if n = 4m.

A square is defined to be *bimagic* if it is magic and the square formed by squaring each of its numbers is also magic; the latter will have *magic constant* $C_2 = n(n^2 + 1)(2n^2 + 1)/6$. A square is defined to be *trimagic* if it is bimagic and the square formed by cubing each of its numbers is also magic: the latter will have magic constant $n^3(n^2 + 1)2/4$.

A *non-simple magic square*, of order n, uses the integers from a specified starting value, s, up to s + n^2, where s=2,3,4... The magic constant is given by $n(n^2 + 1)/2 + n(s-1)$.

Problem 1 Describe and implement an algorithm to construct a magic square with given order, n, and starting value, s.

Problem 2 *Attempt* to establish and implement an algorithm to construct *bimagic, trimagic* and so on squares.

Problem 3 Consider the construction of magic squares whose elements are: (a) Prime numbers (Fig 1); (b) Consecutive prime numbers. Can anyone better the order three magic squares of consecutive ten-digit primes found by Henry Nelson of California?

A *multiplication magic square* has its magic constant, M, defined by the continued product along any row, column or principal diagonal. For example:

```
18   1  12
 4   6   9     M = 216.
 3  36   2
```

A *geometric magic square* is obtained using elements which are a given base raised to the powers of the corresponding elements of a magic square — it is clearly a multiplication magic square. For example, from

```
8   1   6
3   5   7   C=15,
4   9   2
```

and base 2, obtain

```
256    2   64
  8   32  128
 16  512    4
```

where $M = 2^{15} = 32768$.

A *heterosquare*, of order n, is defined to be a square array containing the integers from 1 to n^2 and such that *all* rows, columns and principal diagonals have *different* sums.

A *talisman square*, of order n, is defined to be a square array containing the integers from 1 to n^2 and such that the difference between any integer and each of its neighbours (either row-, column- or diagonal-wise) is (in modulus) greater than some given constant, D. For example:

```
 5  15   9  12
10   1   6   3
13  16  11  14
 2   8   4   7
```

where D = 2. Spencer tells us that: 'The study of these squares is so new, in fact, that no rules for construction are known' . . . neither is there theory to yield D_{max} as a function of n.

Problems 4,(5),((6)) Establish and implement an algorithm to construct multiplication magic squares, (heterosquares), ((talismanic squares)).

Two further thoughts: (i) Show that (excluding rotations and reflections) there are 880 distinct simple magic squares of order 4 and 275305224 of order 5; (ii) Can a talismanic square also be magic?

Attempts at some or all of the above problems may be sent to Mike Mudge, 22 Gorsfach, Pwll-trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel: 0994 231121, to arrive by 1 July 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

101	29	83
53	71	89
59	113	41

C = 213

3	61	19	37
43	31	5	41
7	11	73	29
67	17	23	13

C = 120

Fig 1

Mike Mudge welcomes correspondence on any subject within the area of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

Two children, a boy and a girl, are walking along a street eating fruit. The child eating the apple says 'I'm a girl'; the child eating the pear says 'I'm a boy'. Assuming at least one of them is lying, what is the boy eating?

Prize Puzzle

Regular readers of this column may have deduced that I'm not a great fan of crypto-arithmetic puzzles since I've never before included one. However, there's always a first time, and I rather like this one, so I'll try it for this month's prize puzzle. It should yield to a micro-whirring method of solution, so get the programs written and running:

DAD and MUM are two relatively prime numbers (that is, they have no common factors). When I tell you that DAD/MUM gives the recurring decimal:

.LOVELOVELOVE.......

you should be able to tell me what digit each letter represents.

Answers on postcards or backs of sealed envelopes — no letters please — to: May Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than the last day of May 1991. Good Luck!

Winner, February 1991

A good entry for this puzzle, which obviously set the micros moving. From the 140-odd (some of them very odd indeed!) entries we randomly chose our winner, who is Mr J Lee of Walthamstow, London. Congratulations Mr Lee, your prize is on its way. The three required answers are: 93084, 548834, and 9926315.

To all the not-so-lucky ones, don't give up — it could be your turn next. ■