There's double the mathematical trouble this month as Mike Mudge explores Sequences of Safe Primes and Long Aliquot sequences.

The subject areas for investigation this month have been suggested by Ivo Deütsch of Osnabrück, Germany. They have been visited before during the previous one hundred 'Numbers Count' articles but in each case viewed from a different standpoint.

Area I Sequences of Safe Primes (for an earlier investigation of Safe Primes see *PCW*, September 1990).

Given any positive integer x, define a sequence s_n for n = 0,1,2,3 . . . thus: $s_0 = x$, $S_{i+1} = 2 * s_i + 1$.

Investigate runs (s_0, s_1, s_2, . . . s_k) which consist only of Prime Numbers. In other words, how far can we iterate the Safe Prime Process and only obtain safe primes?

For example, when x = 5, the run extends to k = 3, *viz* 5,11,23,47 before a non-prime (or composite) integer, 95, is generated.

For example, when x = 179, the run extends to k = 4, *viz* 179,359,719,1439,2879 before the non-prime integer 2879 * 2 + 1 = 5759 = 13 * 443 is generated.

Ivo claims a world record with the case k = 6. Can any *PCW* readers equal, or beat, this record?

By examining, systematically, x = 2,3,5,7,11,13 . . . that is, the sequence of prime numbers, is it possible to obtain necessary conditions on x for the run to have a certain length?

Finally, is it possible to prove theoretically that there are runs of arbitrary length? (Are there indeed an infinity of safe primes?) *Note* Assuming conjecture (H) from Sierpinski's book, *Elementary Number Theory*, leads to an expectation of arbitrarily long runs of safe primes.

Area II Long Aliquot Sequences (for an earlier investigation of Aliquot Sequences see *PCW*, October 1983).

For a given positive integer, n, we define its Aliquot Sequence t_i for i = 0,1,2,3,,, thus: $t_0 = n$, $t_{i+1} = s(t_i)$ where s(k) denotes the sum of all the integer divisors of k which are less than k.

Usually, an Aliquot Sequence becomes periodic, though it has not been shown that this is always the case (this is the famous Catalan Problem). For instance, if k is prime, then s(k) = 1, and s(1) = s(0) = 0. If k is *perfect* (see *PCW*, October 1983) then s(k) = k and the constant term will be k from then on.

Of interest are those numbers which generate a long sequence, where 'long' means that it is not periodic after, say, 100 iterations. There are only six starting numbers below 1000 for which it is believed to be undecided whether their Aliquot Sequence terminates — that is, is periodic, the smallest one being 276. What are the others?

If one pursues Aliquot Sequences, peculiar things may happen. Many multiples of 138 or 60 generate long sequences, for example try $2^k * 138$ for k = 0,1,2,3,4,5. Why is this?

In general investigate the lengths of the Aliquot Sequences (within periodicity) of the positive integers n = 1,2,3,4,5,6,7,8,9, and so on. *Note* The primes (and indeed the perfect numbers) are trivial as indicated above; are there other, easily identifiable types of integer for which this is true?

Attempts to investigate one, or both, of the above areas of empirical number theory, together with any other related references or observations may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel 0994 231121 to arrive by 1 September 1991.

Review, January 1991, Beggar My Neighbour

Relating as it does to a pack of playing cards, this topic was popular (despite a somewhat unfortunate misprint — Test Data B Player 2 should have read: Q.J..J.....A.J.....AQ....K).

Robert Newmark discusses project B as starting at 38 card-turns, rapidly rises to a peak at 48 and then falls with very few lasting over 2000, test data B providing the only example over 3000. Number of possible games at 52!/(36!x4!x4!x4!x4!).

This number, about 6.53534135E+20, was confirmed by many correspondents including Frank Webster who ran 100,000 games on an Acorn Electron programmed in BBC Basic and assembler in 15½ hours with mean game length of 256 but a *maximum of 3155.*

As part of an extended contribution, Gareth Suggett provided a reference to a book by Beasley referring to the game as 'Driving the old woman to bed' . . . no doubt frowned upon under the equality of the sexes legislation.

John McCarthy used his 'ageing' QL to play 10,000 games in two-plus hours with a maximum length of 1788 plays with a peak in the interval 40 – 140 . . . somewhat inconclusive.

This month's worthy prizewinner is Ed Hersom of Glen Cottage, Bagby, Thirsk, North Yorkshire YO7 2PF. Ed 'enjoyed playing my thousands of games over Christmas!' He used an RML Nimbus, which has an 80186 chip running at 8MHz (and an 8087) programmed in Forth. This combination played about 16,000 games in three quarters of an hour and thus would take some 200 centuries to play all the available games. Ed took great care to examine his random number generator, attributed to Rickenbacker (*these are often a source of problems in simulation routines*) and based upon what I refer to as a Lehmer Congruence, seed = (seed × 259 + 3) modulo 32 k; this limited Ed's runs to 16,000 games.

LEISURE LINES

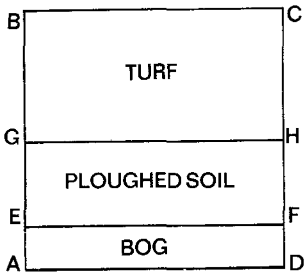
Brainteasers courtesy of JJ Clessa.

Quickie

No answers, no prizes. My friend tells me that her mother is older than her grandmother. And when I checked, I discovered that she's telling the truth. How can it be?

Prize Puzzle

This month's prize puzzle can be solved by simulation, letting the micro do the trial and error stuff.



The diagram represents a square field of 1km side. The field consists of three types of surface:

1 AEFD is bog — AE is 200m wide.

2 EGFH is ploughed soil — EG is 300m wide.

3 GBCH is turf — BG, of course, is 500m wide.

A farmer can travel at 1m/sec in bog; 1.5m/sec in ploughed field; and 2.5m/sec on turf.

What is the shortest time (to the nearest second) in which he can cross the field from point A to the opposite corner C?

Just write the answer on a postcard or the back of a sealed envelope (no letters, please) and send it to July Prize Puzzle, *Personal Computer World* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG.

Winner, April 1991

The winning card came from County Cork in Ireland, from Mr Roger Moran. Mr Anson was the Doctor with an annual salary of £22,596 (or £1883 per month). ■