Mike Mudge proposes a simple divisibility problem using Fibonacci-type sequences.

This research area has been suggested by James S Schofield of Benfleet, Essex who states that it occupied his son and himself on and off for several years before they found a solution. *PCW* readers are invited to reproduce James's solution, to seek out others, to discover some underlying mathematical theory and finally to generalise the problem in a natural way.

Consider the *Fibonacci-type sequence* defined by the simple recurrence relationship: $n_k = n_{k-1} + n_{k-2}$ for k = 2,3,4, ... The full definition of such a sequence requires the specification of the two 'seed' values, n_1 and n_2.

Note $n_1 = n_2 = 1$ defines the Fibonacci Sequence (for further problems associated with this sequence see *PCW* May 1983).

James invites us to consider the case where $n_1 = 1$ and $n_2 = 3$. This generates the Schofield Sequence in Fig 1 (we write $n_k = \underline{S}_k$).

Fig 1

S_k	1,	3,	4,	7,	11,	18,	29,	47,	76,	123,	...
k	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	....

Now divide each term in the Schofield Sequence by its corresponding k-value and establish the remainder. Mathematically the calculation is that of the residue of S_k modulo k. Denote this by R_k, where clearly $0 \leq R_k < k$. See Fig 2.

Fig2

R_k	0,	1,	1,	3,	1,	0,	1,	7,	4,	3,	....
k	1,	2*,	3*,	4,	5*,	6,	7*,	8,	9,	10,	....

The calculation of the last entry in Fig 2 being: $S_{10} = 123 = 12 \times 10 + 3$, thus $R_{10} = 3$.

*Now, it appears from the very small amount of evidence displayed here that whenever $R_k = 1$ then k is a *prime number*!

James sought out, and found, the first occasion on which this conjecture breaks down; he used Computer Concepts' Fast Basic on an Atari 1040 and encourages readers with the thought that the first 1000 primes are sufficient, although the corresponding S-value is found to have 150 digits.

Attempts to find the above number together with any larger solutions are invited, together with possible underlying mathematical theory, and generalised investigations covering a range of (n_1, n_2) pairs. Such attempts may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel 0994 231121, to arrive by 1 October 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained, together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

Review, February 1991, A Problem with Babylonian Fractions

This proved to be a popular subject area, the ingenuity of readers being demonstrated by Ed Hersom, who generated a sequence of 'irrational' numbers for use in problem 2 as sin(2k+1)° for k = 0,1...44.

TDR Redman (and others) corrected the title to Egyptian Fractions and provided a most detailed and interesting early history of same. In the field of applications, NV Meeres reflects the majority opinion with the statement that 'the Babylonian expansion seems unlikely to have any advantage over the continued fraction expansion'.

Gareth Suggett refers to the long list of references in Richard Guy's book *Unsolved Problems in Number Theory* and also to the paper 'On the Adequacy of the Egyptian Representation of Fractions' by P Ernest in the *IMA Bulletin*, October 1980, vol 16, pp219-221.

Among many other results Reg Bond invites readers to verify: 11/178 = 1/17 + 1/337 + 1/145681 + 1/29711989585 + 1/22digits + 1/22digits + 1/43 digits.

The proposer, Nigel Backhouse, has made a major contribution to this area but the worthy prizewinner is TDR Redman of 26 Heathcote Close, March, Cambridgeshire, whose algorithm, written in Integer Basic 2, allows 200 attempts (or 'assaults') on a given fraction in between 27 & 55 seconds on a BBC model B. The most stubborn, needing 49 attempts, being 53/65535 = 1/1241 + 1/347480 + 1/22325590 + 1/267907080.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

No answers, no prizes. Using the digits 1 to 7, each digit once only, make an addition sum which will give a total of 100. For example, you could try 17 + 32 + 4 + 65, but this would not give the required total.

Prize Puzzle

A bit of a number cruncher, this month, so get the programs written and the micros whirring. Three shipwrecked sailors find themselves on a desert island where the only edible items would seem to be coconuts. They decide to gather all the nuts possible and share them out equally. They split the territory into three equal areas and each man puts all the nuts he can find in his area into a pile. When all the nuts have been gathered, they count them and find that the number of nuts in the piles is in arithmetical progression — that is, the difference in numbers of nuts between the second and largest piles is the same as that between the smallest and the second piles. Also, the sum of the numbers of coconuts in any two piles is a perfect square. In addition, the total number of nuts can be divided exactly between the three of them.

Assuming each pile contained at least one nut, what is the least number of coconuts that there could have been?

Write the answer on a postcard, or the back of a sealed envelope (no letters, please) and send it to August Prize Puzzle, Personal Computer World Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 August 1991.

Winner, May 1991

This puzzle was a bit more difficult than usual, or perhaps, like me, most readers do not care for the crypto-arithmetic type of puzzle. Anyway, all the entries had the correct answer which was:

242/303=0.798679867986 . . . . or

A=4: D=2: E=6: L=7: M=3: O=9: U=0: V=8

The winning card, following the random selection, came from Norway, no less, from Mr Gjermund Vage of Trondheim. Congratulations (or the Norwegian equivalent) Gjermund, your prize will be on its way shortly.

As usual, all the rest get our sympathy and our little pep phrase — keep trying, it could be your turn next. ■