Mike Mudge peers round the corners of Integral Triangles and Related Integral Triangles (RITs, ARITs and GRITs).

This research area has come to *PCW* via BG Anderson of Luton, from David Salkeld who is studying the crystalline nature of metals at Hong Kong University. It is an area believed to be 'heavily reliant on computer-based solutions', which has a minimal of mathematical/ algebraic background, and should therefore be suited to many readers of this column.

An INTEGRAL TRIANGLE (IT) is a scalene triangle, ABC, each of whose three sides, a, b & c is of integer length; there being no common factor of these three integers.

The internal bisector of angle A cuts BC in D; the lengths of AD, BD & CD being d, e & f respectively. If d,e & f are also all integers there is said to be a RELATED INTEGRAL TRIANGLE (RIT). It can be shown that for a RIT in which b is the longest side: $d^2 = c^2 + (ac/(b+c))^2 - (ac/(b+c)).(a^2 - b^2 + c^2)/a$......(i)

Clearly, if the 2nd and 3rd terms on RHS of (i) are of equal magnitude, then d=c. This requires $a = (b+c)((b-c/b)^{0.5}$ ......(ii)

In fact (iii) forms a basis for generating all RITs. It shows that: $c/b = (Q^2 - P^2)/Q^2$......(iv) whence $b = NQ^2$, $c = N(Q^2 - P^2)$ for N = 2,3,4...

Putting Q = 2,3,4..,;P = 1,2,3,..Q-1 for each Q and N = 2,3,4... for each P,Q until b-c equals or exceeds a generates all possible b & c.

For d an integer, a is always odd, c & d are always even and b is about 50:50 odd and even. Every value of Q generates at least one integer solution for d; as when P=(Q-1), N=Q & d=c: such solutions are known as 'Argilent' RITs of ARITs. There are however solutions for which d ≠ c, known as 'Grantlie' RITs or GRITs, which fall into one of three categories:

A) GRITs having the same value of P,Q & a as an ARIT: eight are known linked with five ARITs.

eg. (a,b,c) = (97,98,96),(97,343,336),(97,1274,1248);
GRIT, ARIT,GRIT.

Problem A Investigate the remaining (six?) GRITs of this type.

B) Pairs of GRITs with the same a-value and no ARIT linkage: four such doublets are known.

eg. (a,b,c) = (2767,2888,2646),(2767,10108,9261).

Problem B Investigate the remaining (six?) GRITs of this type.

C) Single GRITs. 29 are known up to Q = 51, interesting cases include: (a,b,c) = (193,363,216)...Smallest a-value; (271,392,150)..Highest d/c: (2569,2738,2400)..lowest d/c.

Problem C Investigate the remaining (26?) GRITs of this type.

Problem D Consider the predictability of GRITs, remember that ARITs are completely predictable, so are Type A GRITs?

Problem E For the enthusiast! If the EXTERNAL BISECTOR of A meets CB produced in G where AG & BG have integer lengths g & h respectively, only four solutions are known for integer a,b,c & g.

eg. (a,b,c,g) = (49,875,840,840),(161,3726,3680,12420).

What are the others?

Answers to some of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthern, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 December 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, April 91: A result due to Shrader-Frechette

G Suggett and others submitted proofs of the Shrader-Frechette result which formed the basis of this investigation. TDR Redman examined up to (599,L(q) = 299,i(q) = 2, summultiples 137,162).

An anonymous writer from GL51 (please phone me!) extended this to (991,L(q) = 495,i(q) = 2, summultiples 239,256.

The prizewinner this month is Nigel Backhouse of 64 Bates Lane, Helsby, Cheshire WA6 9LJ, whose extension to (3271,L(q) = 1635,i(q) = 2, summultiples 804,831) was accompanied by a exposition of the relevant theory. Nigel also poses possible extensions and tabulates integers with short repetend length... details on request.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for further Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This month's quickie

An easy quickie — no answers, no prizes given.

A cupboard contains 40 pairs of gloves — 10 pairs each of white, yellow, and blue colours. If the room is in darkness and you want to be sure of getting a matching pair, what is the least number that you must take?

Prize Puzzle

Nov 91

A certain number contains ten different digits. When the number is divided by 9, the result is palindromic. What is the number?

Answers on a postcard, or sealed envelope (no letters please). Send to November Prize Puzzle, *Personal Computer World* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 November 1991. Good luck!

Winner, June 1991

We goofed, it seems. There was not just one, but 13 numbers less than 1000 which do not become palindromic after the stated treatment has been applied at least 1000 times. It serves us right for not checking the result first hand (which we usually have to do, since most of our puzzles are original). Naturally, we allowed any of these 13 as winning solutions, although the one we intended was the least of these — 196. The possible solutions are: 196, 295, 394, 689, 887, 788, 879, their reversals, and 790.

The winning entry, randomly picked, came from Mr RW Newmark of Sunderland, who receives our congratulations (awa' the lads) and a fine prize.

Winner, July 1991

The optimim time needed to cross the fields is 806 seconds (13 minutes, 26 seconds), comprising: Bog 71 secs, Soil 173 secs, Turf 756 secs.

The winning card was sent from London by Ms Klitos Kyriacou. Congratulations, Klitos, your prize will shortly be on its way to you.

Winner, August 1991

Most readers got the correct result of 10,086 coconuts in all, with the three piles containing 482, 3,362 and 6,242 nuts respectively.

The winning card came from Mr Philip Reeve of Huntingdon who gets our congratulations now, and a fine prize shortly.